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The Necessary Redirection of Mathematics, Including Its Relation to National Defense[1]

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Introduction. For years, a deep-seated controversy has been rending education into hostile camps and mutually opposed pressure groups. In this battle, mathematics has been receiving a peculiarly negative treatment. The incredible thing has happened that the earliest and most basic of all the sciences has been relegated to the rank of a minor factor in the secondary curriculum. At least twenty of our states no longer require an hour's study of mathematics in their secondary schools. Arithmetic is being "stepped up," junior high school mathematics has been devitalized, and algebra and geometry are being postponed or omitted. We have been told by a dominant group of educators that, if mathematics is to survive as a school subject, it must be "socialized," that it must stop being "academic," but must become "economically useful." Above all, it must be "adapted to individual needs and interests," and must give its primary attention to immediate "functional" values.

No one will deny that important reforms are necessary in our mathematical curricula and in our teaching procedures, exactly as in all other subject-matter fields. Is that a sufficient cause, however, for the wholesale dropping of mathematics from the educational programs of our young people? The fact that no other country in the world has embarked on such a course should give us pause, but it has had no effect, thus far, on prevailing educational doctrines.[2]

The ruinous effects of our present policies are at last becoming apparent even to the most casual observer. Those who have at heart not merely the status of mathematics in our schools, but more particularly the welfare of our boys and girls and, above all, the safety of the nation, can no

[1] Based on a report presented at a meeting of the Mathematics Club of Rochester teachers of mathematics, on March 17, 1942.

[2] An examination and critique of the educational backgrounds underlying this whole picture, with particular reference to mathematics, has been presented by the writer in a number of studies. See, especially, the following reports:
(1) "The Present Situation in Secondary Mathematics," in The Mathematics Teacher, December, 1940.
(2) "Mathematics as a Universal and Permanent Element in Education," in *Teachers College Record*, November, 1937.
(3) "The Reorganization of Secondary Education," in *The Eleventh Yearbook of the National Council of Teachers of Mathematics*, Teachers College Bureau of Publications, 1936.

longer look upon these developments with resigned indifference. On the contrary, the time is at hand for absolutely frank appraisals of the theories and practices that have brought about the breakdown of mathematics, and for vigorous efforts at correcting the situation in which we now find ourselves. The present paper aims to be a contribution in that direction.

1. Mathematics in the World Crisis

The march of events has an irresistible way of exposing erroneous adjustments and trends. A great world crisis has now drawn us into the most terrific struggle of all time. From coast to coast, in countless shops, factories, laboratories, evening classes, and emergency courses the cry has already gone up: "For God's sake, *teach mathematics!* Let us train *at once* that army of mechanics who must build our airplanes and battleships, make our munitions, and keep open our supply lines! Let us provide *quickly* the necessary thousands of engineers and scientists who shall have the technical knowledge and skill requisite in our gigantic war preparations! Let us create *immediately* an inexhaustible reservoir of competent manpower, officered by men who have a dependable knowledge of ballistics, of sea and air navigation, and of mechanized warfare!"

What mathematics has to do with our war effort has been set forth in considerable detail in the various bulletins or reports of the "War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America."[3] For our purposes we shall submit for perusal only the following passage from the sections which contain the Committee's "Recommendations concerning the field of secondary mathematics." To quote:

"We recommend that, in connection with emphasis on so-called *socialized* aspects of secondary curricula, *a liberalized definition of socialized mathematics* should be adopted *for students of all ability levels, in contrast to more narrow definitions which give unique prominence to business applications and consumer interests.* In the liberalized definition we would emphasize that content with military, scientific, professional, and industrial uses is of a most socialized nature. Also, from the standpoint of a high school student of intelligence, *classical mathematical content may be very 'socialized,' in a true sense,* even though the content possesses only delayed utility, as contrasted with immediate utility in the student's experience."

Do our school courses and procedures in mathematics conform to these recommendations? As a rule, they do not! Instead, we have seen a widespread weakening of mathematics, all along the line. Under the cloak of arresting titles, there has been an ever growing reduction of the "classical mathematical content," with a continuous elimination of algebra, geometry, and trigonometry. The consequences, long foreseen, are at hand. The mastery of even the basic essentials of mathematics is a cumulative task which takes time. It cannot be improvised at a moment's notice, or "picked upon the job when and if needed."

In this hour of extreme peril, military men have found it necessary to tell the plain truth about the nationwide mathematical collapse. None other than Admiral Nimitz has recently supplied grave findings such as the following:

"A carefully prepared selective examination was given to 4,200 entering freshmen at 27 of the leading universities and colleges of the United States. *Sixty-eight per cent of the men taking this examination were unable to pass the arithmetical reasoning test. Sixty-two per cent failed the whole test, which included also arithmetical combinations, vocabulary, and spatial relations.* The majority of failures were not merely borderline, but were *far below passing grade.* Of the 4,200 entering freshmen who wished to enter the Naval Reserve Of-

[3] See, especially, the "Progress Report" published in The Mathematics Teacher, November, 1941; also, the bulletin entitled "Mathematical Education for Defense," reprinted from the *American Mathematical Monthly*, June-July, 1941. Copies of the latter document may be obtained, at $1.25 per 25 copies, from Professor J. R. Kline, University of Pennsylvania, Philadelphia, Pa.

cers' Training Corps, only 10% had already taken elementary trigonometry in the high schools from which they had graduated. *Only 23% of the 4,200 had taken more than one and a half years of mathematics in high school.*

This same lack of fundamental education presented and continues to present a major obstacle in the selection and training of midshipmen for commissioning as ensigns, V-7. Of 8,000 applicants—all college graduates—some 3,000 had to be rejected *because they had had no mathematics or insufficient mathematics at college nor had they ever taken plane trigonometry.* Almost 40% of the college graduates applying for commissioning had not, during the course of their education, taken *this essential mathematical course.* . . .

The experience which the Navy has had in attempting to teach navigation in the Naval Reserve Officers' Training Corps Units and in the Naval Reserve Midshipmen Training Program (V-7) indicates that *75% of the failures in the study of navigation must be attributed to the lack of adequate knowledge of mathematics.* . . .

It is to be noted that the proficiency in arithmetic in the eastern part of the country was strikingly greater than that of the middle west and west. The lowest average mark east of the Mississippi was equal to the highest average mark west of the Mississippi. The three highest average attainments in arithmetic were achieved by the recruiting stations in Troy, Brooklyn, and Buffalo—all in New York State.[4]"

Now, to those who have watched the educational scene with some care, findings such as those mentioned above do not occasion surprise. In fact, not a few similar reports have appeared from time to time. They have regularly been ignored or minimized by those in authority. It is to be hoped that the national crisis will cause a different reaction.

The reasons for the breakdown of mathematics are perfectly obvious. Thus, in arithmetic we have seen (1) a demand by a powerful pressure group to postpone the subject, even as late as the seventh grade; (2) the attempt, in many schools, to introduce the fundamental skills "incidentally," in connection with various "activities" or "centers of interest"; (3) a drastic reduction of all the basic techniques; and (4) an almost universal dependence on mechanical drill, devoid of insight and real mastery. It has become good doctrine to restrict computation to its use in the simplest household and business transactions. As to fractions, according to some "authorities," it is useless to bother a child with anything beyond halves and fourths. The same "experts" have announced that in everyday life one needs only a "reading knowledge" of decimals, aside from ordinary "grocery store arithmetic." Why, then, worry about technicalities beyond these points?

In the case of algebra and geometry, the attacks have been much more vitriolic. It is commonly asserted that these subjects are not "functional." Hence they must go. The fanatical dislike for algebra and geometry has led to frenzied outbursts such as the following, contributed by an educator of nationwide prominence:

"When we get rid of present superstitions we shall certainly advise *not over ten per cent* of our students to take algebra (for strictly prevocational reasons) and *we shall actually prevent at least fifty per cent from taking it*, as we should prevent most healthy people from taking quinine or morphine."

The alleged "uselessness" of algebra, geometry, and trigonometry has become part of the standard vernacular of educational lecture rooms. Only a *vocational* emphasis is still to be tolerated. In the language of a well-known professor of education.

"Algebra, geometry, and trigonometry have a justifiable place in the curriculum only when they are necessary portions of *vocational* courses. . . . Outside of their *vocations*, the only mathematics content really needed by the men and women of the city is *applied arithmetic.*"

The loose and erroneous thinking permeating all these utterances has become

[4] Quoted, by permission, from a statement issued under date of November 12, 1941, by the Bureau of Navigation, Washington, D. C. The complete document was reprinted in THE MATHEMATICS TEACHER, February, 1942, pp. 88 ff.

painfully obvious. Mathematics is *not* merely a matter of computation, of juggling a few formulas. Of even greater importance are the generalized habits which are characteristic of mathematics, together with its types of comprehension and appreciation, and its modes of thinking. In fact, it is these elements primarily which are the carriers of the "functional" values of mathematics. And it is this broader conception of mathematics which underlies our entire material civilization, our age of science and industry.[5]

Clearly, we need a new orientation concerning the place of mathematics in education. Great changes have already taken place in the teaching of mathematics. Not all of them have been in the right direction. In numerous instances, hastily conceived innovations are now sweeping away even the most worthwhile structures of yesterday, irrespective of consequences. Confusion and aimlessness are halting our efforts at reform. It is the thesis underlying this report that our troubles will continue until we have the courage and the intelligence to effect an all-around rebuilding of our educational doctrines, as well as a fundamental and comprehensive reconstruction of our mathematical program.

2. The Inevitable Revision of Educational Theories and Practices

This is not the time and place to analyze, once again, the "pros and cons" of our divergent educational philosophies. They have often been expounded, with sufficient clearness, by representative sponsors.[6] It is desirable, however, to trace the connection between our prevailing educational creeds and the dilemma of mathematics in our schools. Thus, if mathematics submits to the doctrine of "socialization," as now interpreted by its most radical advocates, it will be transformed beyond recognition, becoming merely a tolerated satellite of the social studies. And if it insists on maintaining an independent existence in harmony with its age-long development and its characteristic cultural outlook, it is sure to face further severe onslaughts from its many critics and enemies.

For our purposes, it seems sufficient to examine briefly the widely accepted dogma that "the school *is* life, not preparation *for* life." Taken at its face value, it is commonly interpreted to mean that all school work should be based on the pupil's *personal* and *immediate* "experience." To be sure, there is universal agreement today that nothing can take the place of the child's first-hand, direct contact with the world of reality. All basic meanings must be built up in that way. Moreover, the "scientific method," with its emphasis on direct observation and on careful testing,

[5] Fortunately, there is a constantly increasing body of literature dealing with the *cultural* aspects of mathematics, and with the theory of "transfer." These questions are discussed with force and clearness in the new national reports. An excellent addition to this discussion is Dr. B. R. Buckingham's paper on "The Contribution of Arithmetic to a Liberal Education," in The Mathematics Teacher, February, 1942.

Thus far, many educational theorists and curriculum specialists have chosen to ignore this literature. They have even branded it as a futile "defense mechanism," or as the last, despairing outcry of the "vested interests." Until this attitude is corrected, it will be impossible to effect a greater degree of mutual understanding and coöperation.

[6] For example, brief accounts of our contrasting philosophies of education appeared in recent issues of *The Journal of the National Education Association*, as follows:

(1) Bagley, William C., "The Case for Essentialism in Education," October, 1941.
(2) Kilpatrick, William Heard, "The Case for Progressivism in Education," November, 1941.

An authoritative review of Professor Dewey's lifework as a philosopher and an educational thinker is now available. See, Schilpp, Paul A. (Editor), *The Philosophy of John Dewey*, Volume I of the Library of Living Philosophers, Northwestern University, 1939. This volume contains a complete bibliography of Dewey's writings (1882–1939), in a section comprising 65 pages, as well as 17 descriptive and critical essays on the philosophy of John Dewey, "by leading exponents and opponents of the philosopher's thought." Of particular importance to teachers, in this symposium, are the contributions of Childs, Kilpatrick, and Whitehead.

is largely responsible for the modern era. But does all that justify the extreme views now dominating education?

Schools which profess their faith in an "experience curriculum" aim to readjust their programs at a moment's notice, in accordance with the latest "findings." They are really planless. To them, the world is a moving picture, with a continuous performance that may suddenly bring to light strange and unexpected revelations. Is not this the basis of our ceaseless and yet so barren attempts at curriculum revision? With ever new "objectives," ever more startling "approaches," and countless "criteria," we seem to be moving on a merry-go-round that gets us nowhere. In the language of a witty critic, we appear to have chosen St. Vitus as the patron saint of education.

It is constantly overlooked, of course, that if mankind, in its age-long struggle, has learned little that is worthy of attention, then surely a few years of personal adventuring in a Progressive school, or in daily life, must be equally futile. In other words, the doctrine of "immediate experience," when given the current interpretation, leads to educational chaos.

Now, the study of mathematics demands, above all, constant attention to *order*, *direction*, and *form*. It cannot be fitted to situations which lack these elements and are characterized merely by variety. Hence much of the wrath now directed against mathematics. That a good deal of this fury is really uninformed makes it all the more tragic.

Able writers have effectively exposed other weak spots in the current educational philosophy. For example, certain overworked dualisms, as Bagley and others have insisted right along, are purely artificial. It is not a question of interest *versus* effort, of immediate *versus* ultimate objectives, of "functional" *versus* deferred values, of "integration" *versus* subjects, and so on. The obvious fact is that in each case we need *both* ingredients of these alleged opposites.

The rebellion against our erroneous educational trends has been gathering force for some years. The world crisis will greatly accelerate the house-cleaning process. For the demand of the hour, once again, is *Efficiency*. And that, as Morrison has been saying so emphatically, has most certainly been lacking in our educational enterprises. The "child-centered school" has been found wanting and seems to be on its way out. Its successor, the "society-centered school," will fare no better unless it develops a real understanding of the objective realities of life, stresses organized and tested knowledge, and instils a greater degree of genuine discipline.[7]

In the coming reconstruction of education, mathematics is destined to occupy a much more important place in the curriculum, because this age of science, industry, and commerce is "incurably mathematical," and because many of the affairs of everyday life can neither be *understood* nor properly *controlled* without the guiding influence of mathematics.

3. The Theory of Life Situations and the Concept of Social Utility

Teachers of mathematics have made it extremely easy for their adversaries to find fault with their subject and their procedures. Even the most ardent friends of mathematics have long admitted that a tremendous change is necessary in the training and the equipment of our teachers, in classroom practices, and in the spirit of the profession. Thus, more than ten years ago, Professor Charles H. Judd predicted the breakdown of mathematics as a school subject, unless the teachers developed a different attitude.[8]

[7] For a masterful critique of present educational trends, see Breed, Frederick S., *Education and the New Realism*, The Macmillan Company, 1939.

Particularly revealing, also, is a recent article by Louis Foley, on "The Philosophy of the New Education: A Reply to Professor Kilpatrick," in *School and Society*, January 10, 1942.

[8] See, especially, the basic paper presented by Professor Judd to the National Council of Education, February, 1927, on "The Fallacy of

Back of this picture, of course, is the prior responsibility of our normal schools and colleges, of examining boards, of school authorities, of backward curricula, of textbook writers, of hoary traditions, and so on. A Herculean job of all-around reconstruction and improvement lies ahead of us.

Aside from many minor faults, the chief defect in current mathematical teaching, according to our critics, is that it is "out of step with Life," and that it has become an array of "inert ideas." None other than Professor Alfred N. Whitehead, the distinguished mathematician and philosopher, has furnished the most compact formulation of this criticism. To quote,

"There is only *one* subject-matter for education, and that is *Life* in all its manifestations. Instead of this single unity, we offer children—Algebra, from which nothing follows; Geometry, from which nothing follows; Science, from which nothing follows; History, from which nothing follows; a couple of Languages, never mastered. . . . Can such a list be said to represent Life, as it is known in the midst of the living of it?[9]"

This passage was written *thirty years ago!* It was recently chosen as a central motto by the Educational Policies Commission for its widely publicized volume on *The Purposes of Education in American Democracy* (1938).

Not a few of the world's leading mathematicians on both sides of the Atlantic have expressed, again and again, views similar to those of Professor Whitehead. Their wise counsel was not heeded. Thus, the recommendations contained in Professor E. H. Moore's famous presidential address of 1902 anticipated by *forty years* the essentials of the indispensable mathematical reforms that even now we seem unable or unwilling to undertake. How many normal schools and colleges pay the slightest attention to these things, especially in their teacher training courses?

But in all this there is still another note of grim irony and accusation. For it is quite certain that man's first steps in mathematics were occasioned by fundamental human needs, as were all the other early manifestations of human culture. The basic mathematical concepts, principles and techniques still reveal countless traces of the soil from which they sprang. The decimal system is forever linked to the fingers, and "calculation" will always remind us of the long ages of pebble-counting, including the abacus. The very word "geometry" means "earth measurement." Human ingenuity has created no more universally applicable tool than mathematics. Number and form, counting and measurement, were indeed among the most priceless weapons in man's epic struggle for existence.

And so, to any one who has perused the story of mathematics even superficially there is nothing surprising in recent demands for a "life-situation approach." Why, then, a perplexed teacher may ask, should we not greet with unrestrained enthusiasm this well-nigh overwhelming crusade on behalf of a more "life-like" and "socially useful" type of mathematical instruction?

Who can deny that to depict "Life in all its manifestations" in the schoolroom, to incorporate it adequately in textbooks and daily lessons, would require the wisdom of the ages and the skill of superteachers. But aside from that, two major impediments have regularly blocked the creation of a "life-centered" mathematical program.

(1) The range of potential mathematical applications is of encyclopedic dimensions. The choice of a nationally significant and authoritative group of life situations, planned systematically for school pur-

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Treating School Subjects as 'Tool Subjects'," published in the *Third Yearbook of the National Council of Teachers of Mathematics*, 1928.

[9] The statement quoted above was included by Professor Whitehead in an address which he delivered in 1912 before the Educational Section of the International Congress of Mathematicians, at Cambridge, England. This address is now Chapter I of Whitehead's *The Aims of Education and Other Essays*, the Macmillan Company, 1929. It is a volume which still deserves the careful attention of every teacher of mathematics.

poses, clearly transcends the competence of both local curriculum committees and individual authors, however sincere and industrious they may be. For a considerable period, this problem will demand intensive study, controlled experimentation in many laboratory schools, and a nation-wide coöperative effort.

(2) The present time allotment in secondary mathematics is totally inadequate even now, being far below the *minimum* considered necessary in other countries. The wholesale injection of all sorts of life-situation backgrounds into such a crowded curriculum can have but one result, that of displacing or greatly reducing the very moderate amount of real mathematics we now try to teach. The disappearance of algebra and geometry which we are witnessing everywhere proves the correctness of that statement. The only remedy would be that granted so freely now to the social studies and the science programs. It consists in a curriculum extending continuously from the first school year to the twelfth.[10] Only then could we cope successfully with the absolutely essential mastery of the basic mathematical techniques, with the modes of thinking that are so vital for "transfer," and also with desirable "functional" applications. A few unrelated snapshots of "social" or "community" problems, here and there, will satisfy neither the friends nor the foes of mathematics.

In short, until we can provide for both *system* and *continuity*, there can be no adequate solution for the life-situation approach in mathematics. It would be possible to support this position at every point by extensive quotations from the writings of both Progressive and conservative educators. Dewey himself has made an urgent plea for systematic and sequential learning.[11]

We have spoken of adequate criteria of selection. In particular, just what constitutes "social utility?" Surely we have recovered from the kindergarten disease of twenty years ago when it constituted a favorite pastime to list the "frequencies of occurrence" of proposed curriculum items in newspapers and periodicals, and to gauge their "social importance" largely by that inadequate criterion. And we have long discounted the usual lists of "minimum essentials," as well as alleged inventories of "adult" of "consumer" usage, nor are we swept off of our feet, any longer, by the uncritical wave of enthusiasm that seemed to create such a sudden interest in "consumer" themes like installment buying, owning and operating a car, reading gas meters, using time tables, and the like. Today we realize that "social utility" must take into account not only the present, but also the future; not only *actual* usage, but also *desirable* usage; not the lowest levels of intelligence, but the highest probable average; not merely the followers, but also the leaders.

We shall close these considerations by calling attention to a truly significant statement bearing on this entire issue. Its author, Dr. Harold Rugg—himself a pas-

[10] Considerable support has been given to the demand for greater continuity and coherence by the recent national reports on the teaching of mathematics.

The Fifteenth Yearbook of the National Council of Teachers of Mathematics represents the Final Report of the Joint Commission on *The Place of Mathematics in Secondary Education*, Bureau of Publications, Teachers College, Columbia University, New York, 1940. It offers a six-year curriculum in secondary mathematics, extending from the seventh grade through the twelfth. (This report is sometimes referred to as the JCR.)

The Sixteenth Yearbook of the National Council of Teachers of Mathematics contains the Final Report of the National Council Committee on Arithmetic, entitled *Arithmetic in General Education* (1941). It stresses the fact that arithmetic is a "system" of ideas.

Of particular interest, in this connection, is the Report on *Mathematics in General Education*, prepared under the auspices of the Progressive Education Association, D. Appleton-Century Company, New York, N. Y., 1940. (This report will be referred to as the PEAR.)

[11] See Dewey, John, *Experience and Education*, The Macmillan Company, 1938, p. 111; also, "Democracy and the Curriculum," the *Third Yearbook of the John Dewey Society*, D. Appleton-Century Company, 1939, pp. 414 ff.

sionate advocate and perhaps the best informed student of "social analysis"—nevertheless urges these important precautions:

The task of stating the goals of education is not to be consummated by an analysis of social activities alone. It will be aided by such an analysis, but must not be dominated by it. It will be achieved only by hard thinking and by the most prolonged consideration of facts by the deepest seers of human life.

Social analysis tells us merely what techniques and what specific kinds of knowledge we should have on tap. For the basic insights and attitudes we must rely, as we do for the statements of the goals of education, upon human judgment. It is imperative, however, that we make use of only the most valid judgments. *The forecasting of social trends, the perception of the focal problems and issues, and the connections underlying thsm, demand erudition and maturity of reflection that eventuate only from a prolonged and scientific study of society. To the frontier of creative thought and of deepest feeling we go for guidance as to what to teach.*[12]

4. Can There Be a Life-Situation Approach in Mathematics?

In one sense, this question seems utterly absurd. For mathematics, as we have seen, is the outgrowth of human needs, and its range of practical applications is unlimited. Nevertheless, there are compelling reasons for regarding an honest analysis of this question as the key to the present controversy in the field of mathematics.

First, a very large part of the mathematical edifice owes its existence, *not* to practical or "social" considerations, but to research undertaken for its own sake. And a great deal of this research has turned out to be of crucial value to science, business, and industry. Without a passionate devotion to "pure science," irrespective of utilitarian values, human progress would soon come to an end. It is a grave mistake to suppose that this broad "academic" training can be postponed indefinitely, or that it can be improvised at a later date, or that it should be reserved for a few prospective "specialists." The present acute shortage of trained scientists and research workers, as well as the deplorable situation pointed out by Admiral Nimitz, is correctly attributed to a "deficiency in the early education" of our young men.

Second, an exclusive regard for practical applications tends to rule out, even in elementary mathematics, many essential phases of the subject that cannot be applied at once. What is the *immediate* practical value of the number system of algebra, of the postulates of geometry, of basic modes of thinking, and the like? The complete destructionism lurking in a radical life-situation approach, when thus interpreted, should be apparent to any open-minded person. Ludicrous consequences follow from an out-and-out utilitarianism. The "ultimate objectives" of education cannot be circumscribed by narrowly defined "functional" values.

The serious implications which surround a radical life-situation program in mathematics are certainly not imaginary. On the contrary, they have begun to condition the very existence of mathematics as a school subject. The dubious slogan of "social utility" has already weakened arithmetic and has eliminated algebra and geometry from the school diet of millions of our young people. As a result, the world crisis is finding the majority of them unprepared in the fundamentals which are so badly needed for purposes of national defense. Surely, we have been following a "suicidal" course, as a recent defense report expressed it. A return to sanity is absolutely imperative. What we need is a convincing set of guiding principles which shall enable teachers to appraise more adequately the respective virtues or defects of conflicting educational programs. We shall therefore submit below a series of closely related "propositions" that seem pertinent to this discussion.

To make the problem as simple as possible, we shall exclude from consideration

[12] See Rugg, Harold, *American Life and the School Curriculum*, Ginn and Company, 1936, p. 203.

the complex domain of *vocational* education, and we shall assume the existence of but *two basic curriculum approaches in mathematics*. Thus, the curriculum may be built on organically arranged *mathematical categories* which, however, are constantly motivated and made meaningful by numerous *life-situation backgrounds* and *problems*. We shall call this the $M \rightarrow L$ approach. Again, one may depend entirely on a series of life situations and from them try to obtain the desired mathematical values. In that case we have an $L \rightarrow M$ approach.[13]

With regard to these opposite approaches, then, we now turn to the following "propositions." They are given without "proof," although it would be an easy matter, in each case, to supply an authentic "demonstration."

Proposition 1. Mathematics is a system of closely related concepts, principles, techniques, and modes of thinking.

In support of this proposition, we call attention to the following significant passage from the Sixteenth Yearbook of the National Council of Teachers of Mathematics, on *Arithmetic in General Education* (1941). It applies to the whole field of mathematics. To quote,

"*Arithmetic* [mathematics] *is a system of ideas.* It is not a collection of objects. It is not a set of signs. It is not a series of physical activities. Arithmetic is a system of ideas. Being ideas, arithmetic exists and grows only in the mind. It does not flourish in the world of things. It does not arise out of sensory impressions. *It has nothing to do with the amount of chalk dust forty pupils can raise in a schoolroom in thirty minutes. Arithmetic exists and grows only in the mind.* Being a system, arithmetic must be taught as a system. It is not an outgrowth of the individual's everyday experiences. It is not learned according as the interests or the whims of pupils may suggest. It is not anyone's personal discovery or invention. *Arithmetic* [mathematics] *must be taught as a system.*[14]"

[13] For a more detailed discussion of these two approaches, see THE MATHEMATICS TEACHER, December, 1940, pp. 348 ff.

[14] Op. cit., p. 80.

It is almost incredible that this obvious fact needs to be restated so emphatically. But in the interest of mathematical recovery this is essential.[15]

Proposition 2. Any $L \rightarrow M$ approach tends to be incurably unsystematic.

This means that the mathematical concepts, principles, and skills needed in dealing with a set of life-situation problems such as are being suggested for an $L \rightarrow M$ approach do *not* appear in a sequential, cumulative arrangement. They do *not* follow either a logical or a psychological order. Moreover, they tend to be "bunched," precisely as in most vocational or technical "projects."

Again, an examination of "socialized" curricula or textbooks in mathematics shows that the "life-situation" *backgrounds* receive the primary emphasis, while the *mathematical* ingredients have to "make the best of it." Only too often, even the key concepts are neither explained clearly nor defined with care. Modes of thinking receive little or no attention. Bits of theory are interspersed "when and if needed." "Units" bearing inviting "social" legends often become a confusing assemblage of the most heterogeneous items. The enthusiastic believer in "socialized" or "practical" mathematics seems to be oblivious to such grievous didactic blunders. Nor does it disturb him that the plan imposes even on superior pupils an almost impossible learning job. For slow or retarded pupils it means fail-

[15] A group of independent thinkers have consistently protested against the psychology of "bonds" and "specific" skills, and against the prevalent atomization of mathematics. Thus, Professor Charles H. Judd's views on *generalization* and on *transfer* have been vindicated beyond reasonable doubt. Teachers of mathematics should not fail to re-read and study Professor Judd's trenchant discussions pertaining to *general* verus *specific* training. See, especially, the following works by Professor Judd:

(1) *Psychological Analysis of the Fundamentals of Arithmetic*, The University of Chicago, 1927, pp. 116 ff.
(2) *Psychology of Secondary Education*, Ginn and Company, 1927, Chapter XIX.
(3) *The Psychology of Social Institutions*, The Macmillan Company, 1926, Chapter V.

ure or, at best, a vague impressionism devoid of real understanding or mastery.

Surely, until these fatal defects in the present programs for the "reconstruction of mathematical experience" are corrected, every informed student of mathematical education will continue to ask the critical question, "Whither socialized mathematics?"

Proposition 3. In an L→M approach, the necessary mathematical techniques must be taken care of either by a plan of unrelated mechanical drill or by the road of incidental learning.

This is really a corollary of the preceding proposition. An inspection of current courses and textbooks leaning toward an *L→M* approach will prove that they invariably face this dilemma. That is why the *L→M* approach violates certain cardinal principles of the psychology of learning. Thus, years ago, Professor Dewey condensed a good deal of excellent theory into this key sentence:

> Practical skill, modes of effective technique, can be intelligently, non-mechanically *used* only when intelligence has played a part in their *acquisition.*[16]

In its eagerness to *use* the tools of mathematics, the *L→M* approach pays little or no attention to their systematic development and their *meaningful acquisition.* But there is no doubt that, in the language of Professor Dewey, "use" or "drill," when divorced from insight, is mere "animal training." At a later time Dewey wrote this illuminating statement which also has an important bearing on the problem of incidental learning:

> "Instruction in number is not concrete merely because splints or beans or dots are employed. *Whenever the use and bearing of number relations are clearly perceived, a number idea is concrete even if figures alone are used.* Just what sort of symbol it is best to use at a given time—whether blocks, or lines or figures—is entirely a matter of adjustment to the given case. If the physical things used in teaching number or geography or anything else do not leave the *mind* illuminated with recognition of *meaning beyond themselves,* the instruction that uses them is as abstruse as that which doles out ready-made definitions and rules, for it *distracts attention from ideas to mere physical excitations.*[17]"

Today there is complete agreement that neither exclusive dependence on unrelated mechanical drill nor "incidental learning" will get us anywhere in mathematics.[18]

Even Progressive writers now freely admit the futility of "incidental learning" in mathematics. Thus, in his most recent volume Professor Melvin says,

> "The place of drill in the modern school has been much misunderstood. Some proponents of change have gone so far as to teach that children can learn the number processes incidentally, completely obviating the necessity of drill. Some say that selling stamps in a post office or making change in a school store provides sufficient training so that no drill is needed. Theoretically it may be granted that there are bright children who could learn number thus, on the side—this provided their teachers could bear teaching it by interrupting and cross-cutting more important experiences to dwell upon number. *But schools which have experimented with such attempts have always failed to meet the needs of all the children without drill.* Misunderstanding is further due to failure to realize that drill is a normal part of more extended activities. It is a special activity needed in the support of other activities. . . .

[16] Dewey, John, *How We Think,* D. C. Heath and Company, 1910, p. 52.

[17] See, *How We Think,* op. cit., New Edition, 1933, pp. 224 ff.

[18] A careful psychological critique of the "drill theory of arithmetic" and of the "incidental learning theory," as well as an exposition of the "meaning theory of arithmetic instruction," was contributed by Dr. W. A. Brownell to the *Tenth Yearbook of the National Council of Teachers of Mathematics* (*The Teaching of Arithmetic*), 1935. In the same Yearbook, a revealing study by Dr. Paul R. Hanna and others gives details as to the way in which arithmetic appeared incidentally in an activity program. Another basic monograph on this subject is Chaper IX of the *Sixteenth Yearbook of the National Council.* Reference should also be made, in this connection, to a classic paper by Dr. B. R. Buckingham, on the theme, "Significance, Meaning, Insight, These Three," in THE MATHEMATICS TEACHER, January, 1938.

The key to good number drill is mastery step by step. Arithmetical ability is a complex and integrated skill. The number processes form a hierarchy of habits, each of which depends on the subsidiary habits built up before. Number work is in this respect unique. It therefore requires unique treatment. . . . Thus, throughout the whole teaching of number, the drill should be arranged to enable the individual to secure mastery.[19]"

Again, the Third Yearbook of the John Dewey Society (1939), while strongly condemning "the deadly routine of drill on trivial matters which even now forms no small part of the educational program," nevertheless warns against the doctrine of incidental learning in a highly important passage which reads as follows:

"There is every reason to believe that the *incidental consideration* of the more complex abilities which are used frequently in varying situations tends to result in *ineffectual mastery of them.* It is sound education to see that these abilities are developed as *economically and as effectively as possible.* Adequate curriculum design will recognize this factor. . . .

Abilities are continually required which cannot be mastered on the spot and which may be mastered for more effective future use if given direct emphasis. To stop a unit of work or a project for this direct emphasis frequently disrupts the flow of events and diverts attention from the purpose at hand. This difficulty may be avoided by provision of a time for this direct emphasis as need is noted.[20]"

If all this is admitted, it is certainly permissible to question the efficiency of any plan that makes the systematic mastery of mathematical concepts, principles, and skills a lucky accident instead of an assured certainty.

Proposition 4. A genuine L→M approach would certainly demand a much greater time allowance, as well as more extensive training and greater effort on the part of the teacher.

Every attempt to use an *L→M* approach, *within the usual time limits,* has regularly led either to an overcrowded program, or to a planless and superficial assortment of "odds and ends."

Again, competent students of the curriculum agree that the "social needs" or "experience" approach presupposes a "constructive group attack," as well as *continuous planning* and an *unending program of revision.* But they also admit that all this imperatively demands "adequate educational leadership." To quote,

"Leadership must be ever present, stimulating, guiding, suggesting; leadership which is concerned with procedure and has faith to trust the outcome so long as the procedure is sound. Such leadership is the greatest need in developing curriculum programs which make operative the widespread teacher participation and group action which is basic to curriculum improvement.[21]"

Can there be any reasonable doubt, in view of these considerations, that the organization of an *L→M* approach represents a formidable task which is simply beyond the professional ability of the individual teacher in the average school?[22]

Proposition 5. There appears to be no escape, then, from an M→L approach, particularly if dependable "results" are desired, and if mathematics is to make the cultural contribution of which it is capable.

This view is the one endorsed by the Fifteenth and Sixteenth Yearbooks of the National Council.[23] As to the PEAR, it

[19] See Melvin, A. Gordon, *Method for New Schools,* The John Day Company, 1941, pp. 62 ff.

[20] See the *Third Yearbook of the John Dewey Society,* op. cit., pp. 430 ff.

[21] Ibid, p. 474.

[22] Educational writers of the most diverse schools of thought share the conviction that the *teacher* is the "key" to the problem of curriculum improvement and of educational advancement. "As is the teacher, so is the school."

See, however, Mort, Paul R. and Cornell, Francis G., *American Schools in Transition,* Bureau of Publications, Teachers College, New York, N. Y., 1941. This study points out in a startling way the extreme slowness with which the average school system responds to adjustments which have long been considered "must" propositions by educational leaders. In particular, Chapter XI, on "Teachers as Adaptors," is far from complimentary to the teaching profession as now constituted.

[23] As stated above, the "Joint Commission Report" (Fifteenth Yearbook of the National Council) advocates a six-year program in sec-

presents a very significant set of basic mathematical categories, but it has not yet been implemented by a detailed program showing how the life-situation problems which it advocates may be used successfully in the curriculum of the average school.

The validity of Proposition 5 does not rest, of course, on any committee report.It is firmly anchored in the very nature of mathematics. All competent scholars in this field agree that mathematics is a characteristic constructive enterprise of the mind. This mental activity is essentially "free," not being dominated by "things," except in so far as fundamental *mathematical concepts* and *relations* may be *suggested* by corresponding *percepts*. That is, even the "purest" kind of mathematical thinking does not originate in a vacuum, but receives its basic impulse through its contact with the world of concrete reality.

And so, it is not strange that "pure mathematics" can be widely *applied*. For an illuminating discussion of the underlying theory we may refer, for example, to Professor Cassius J. Keyser's interesting volume entitled *The Pastures of Wonder—The Realm of Mathematics and the Realm of Science* (Columbia University Press, 1929). Of particular importance, in this connection, are the following statements:

"*One can think mathematically about any and every sort of subject-matter* to which one may be drawn by love or driven by the exigencies of life. The fact ought not to astonish, for all subject-matters are but fragments or constituents or aspects of one whole—the world, including man or, if you prefer, included in him. . . .

The validity of mathematical propositions is independent of the actual world—the world of existing subject-matters—is logically prior to it, and would remain unaffected were it to vanish from being. . . .

The applications [of sheer mathematical thinking] admit of endless multiplication not only but of *extension to all subjects*. . . .

Sheer mathematics is essentially and exclusively concerned with the *world of the Possible*. It is *applicable* to the actual world because the *world of the Actual* is part of the possible, else it could not be actual."

Proposition 6. In the case of either approach the choice of significant and worthwhile life situations is a highly technical task which will demand organized research and extensive coöperative testing.

Among those who have studied the situation, there is no difference of opinion concerning the large-scale effort that will be necessary to create a truly functioning "social" emphasis of national scope in our school programs. This view is evidently shared by the makers of the PEAR, for they tell us that

To make the proposed program [of life-situation problems] effective and to supply innumerable details of possible content and organization will require experimentation, both extensive and intensive, over a period of years.[24]

Even more emphatic is the closing remark of an address made by Dr. M. L. Hartung, one of the authors of the PEAR, at the 1939 meeting of the National Council. He said,

To build a curriculum along these lines we must sentence ourselves to many years of hard labor.[25]

At present, utter confusion still characterizes the nation's curricula. Nor do any two textbook writers seem to have a common point of view regarding the use of "social" data and life problems. Analyses of current textbooks show a well-nigh complete discrepancy of emphasis all along the line. An authoritative account of this situation will be found in the recent volume entitled "What Our Schools are Teaching," by Dr. H. B. Bruner and others (Teachers College Bureau of Publications,

ondary mathematics, based on carefully formulated mathematical categories. It submits a grade-placement chart showing how such a trunk-line curriculum can be organized. And it also offers suggestions as to the choice of suitable life-situation problems.

[24] See the PEAR, op. cit., p. 14.

[25] See, "Mathematics in Progressive Education," by Maurice L. Hartung, in THE MATHEMATICS TEACHER, October, 1939.

1941). We are told that not less than 85,000 courses of study are now assembled at the Curriculum Laboratory of Teachers College, and the end is not yet. Only a minority of the courses were found to possess real merit. Of 3,277 curricula in mathematics, only 17% were pronounced "outstanding" by Dr. Bruner's rating staff.

An aimless and often amateurish individualism is destroying the educational solidarity of the nation, at a time when we need such solidarity most badly. Those who desire a return to sanity will derive comfort and strength by reading, for example, Walter Lippmann's address on *Education vs. Western Civilization.*[28]

In the light of the preceding discussion, we ask again, "Can there be a life-situation approach in mathematics?"

Only one answer appears to be possible. For reasons which will now be understood, we feel that it should undoubtedly take the following form:

A life-situation approach in mathematics, especially under existing schoolroom conditions and time allotments, seems feasible only if it is of the M→L type.

Conclusion and Summary. We have tried to point out some of the salient features of the necessary redirection of mathematics. We have stressed the importance of a broader conception of mathematics. A purely vocational or utilitarian emphasis is simply out of the question. Mathematics may indeed be highly "socialized" even when it is based on the "classical mathematical content." We have suggested, however, that every opportunity should be taken to make use of genuine, life-centered applications.

It may have been noted, throughout the discussion, that we have refrained from offering a *definition* of "socialized mathematics." In view of the many interpretations that have been given to it, a unitary explanation seems impossible. Its essence, however, rests on the thesis that the teaching of mathematics can and should reflect more adequately the affairs and interests of everyday life, or "Life in all its manifestations." We have stated that the task of planning the mathematical curriculum along these lines is one of vast dimensions, transcending the capacity of both individuals and local committees. The completely chaotic character of our present curricula and our textbook literature amply proves that point. And so, if real progress is to be made in the "socialization" of mathematics, we must have organized research by regional and national groups of competent students, supplemented by controlled experimentation in numerous laboratory schools. Then, too, to make such a program of "socialization" effective in our schools, we need a continuous twelve-year curriculum, and a far better preparation of our teachers. These are also the conditions necessary for the creation of differentiated programs for the various ability levels which we have in our schools, and for the introduction of those *cultural* aspects of mathematics which are the chief carriers of its potential "transfer" values.

Professor Hogben, it will be recalled, has referred to the history of mathematics as a "mirror of civilization,"—an excellent comparison. But the human scene, as a whole, is too vast for ready comprehension and assimilation. In the classroom we must bring to a sharp focus the *central* mathematical concepts, principles, techniques, and modes of thinking, in a social setting that is not merely appealing, but also authoritative and generally significant. How to combine real understanding and mastery, such as we need in mathematics, with a broad social outlook, is not the simple assignment that some educators seem to think it is.

It is important that we be on our guard against "getting lost" in a welter of unorganized statistics, graphs, formulas, community backgrounds, business and industrial details, consumer problems, and the like. The danger of going astray is very great. Constantly, new fads and magic

[28] An address delivered at the University of Pennsylvania, and published in *The American Scholar*, Spring 1941, Vol. 10, No. 2.

slogans are being paraded before our eyes. We should certainly reject a conception of "socialized mathematics," or "consumer mathematics," or "practical mathematics," or "community mathematics," which amounts to giving up mathematical core ingredients in favor of "adolescent or social needs." We must not wind up by having "mathematics" *without mathematics!* We shall continue to oppose any plan which advocates brief "exposure" or "terminal" courses of one or two semesters. Such courses have been tried many times in the past, and they have always proved unsatisfactory. Why repeat old mistakes again and again? Instead, we must "bring order out of chaos" by *maintaining a genuine sequence, cumulatively built, of those enduring mathematical categories which are the product of the ages and the common warp and woof of all mathematical applications. In short, we can endorse only an M→L approach.*

The national crisis is revealing the fatal weaknesses of our present educational policies. The huge struggle in which we are involved will force us to turn away from an almost pathological individualism to a consideration of the things that should come *first.* The idea that we must have a special streamlined curriculum for every town and hamlet is at last seen to be "not only tragic but silly." Combined with the doctrine of "individual needs and interests" it has destroyed vitally necessary common backgrounds and types of understanding in the lives of the rising generation—an extremely serious menace to the nation's unity and efficiency. Our common perils will compel us to rule out personal whims as the controlling factor in so crucial a matter as the preparation of our youth for the battle of life.

In this crisis mathematics is being called back to the seat of honor from which it had been driven by shortsighted and confused educational counsels. It is coming back because of its age-old service record and because of its enduring and yet ever new *cultural* bearings. It will be reinstated in our school courses, and it will be taught by a body of teachers who will try to be more alert than in the past, as well as more intelligent about the real objectives of mathematical education.

The program which has been suggested in this paper is neither visionary nor futuristic. On the contrary, its underlying principles have been tested, over a period of years, by a sufficiently large number of capable teachers, in representative and widely separated school systems, to warrant confidence in their validity. It is in harmony with the experience of all other leading countries. The evidence is overwhelming that when mathematics is taught as a *cumulative system of ideas, with due regard for understanding and mastery,* and for *life-centered applications,* it ceases to be a meaningless game. Instead, it assumes its rightful place in the curriculum as an ageless element of human culture and as an indispensable factor in national defense. And that is the *Redirection of Mathematics* which we should all help to put into effect without further delay.

WATCH THE May issue of *The Mathematics Teacher* for the Summer 1942 program of the National Council of Teachers of Mathematics to be held in Denver, Colorado in connection with the N. E. A.

"Lord, Deliver Us from Our Friends"*

By JOSEPH SEIDLIN
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THE PLACE of Mathematics in the curricula of our secondary schools has been variously defended and attacked. In other words, mathematics has "friends" as well as "enemies." It seems that the cause of mathematics suffers more from the attempts of some of its friends to save it, than from the attempts of some of its enemies to destroy it.

Not so long ago, I asked a thousand mathematicians for the evaluation of the traditional heart of our secondary school program of mathematics—plane geometry. I have since published their replies—at least such of their replies as were fit to print. Their comments were fairly evenly divided between eulogies and outright denunciations. Many recommended the complete elimination of the subject from the high school curriculum. Please bear in mind that the source of these recommendations is not the social studies group, but mathematicians of high order and rank.

Again, not so very long ago, the Commissioners or Superintendents of State Departments of Education, in reply to a specific and direct query, expressed themselves specifically and directly on the trends in secondary school mathematics. By grouping the mathematicians in class intervals of 15, it were easy to show that a one to one correspondence exists between the recommendations of these two groups.

For the recommendations of the educationists, I have no such objective data. I have been obliged to depend on a somewhat sensitive thermometer which measures the temperature of my ears every time they burn. Only occasionally was I able to check on the correctness of that measurement by a very critical color-discriminator which is capable of measuring and detecting fine shades of red or purple whenever my face turns that way. A great many educationists are quite impartial. They love education well, but not too wisely. To them, the millenium in Education will come when, and only when, all education shall be wholly devoid of all subject matter.

"The people" hold mathematics in deep reverence and respect. They are so impressed with the majesty and nobility of mathematics on one hand, and are so confident of their children's mental prowess on the other hand, as to become much disturbed, distressed and even enraged, when the twain fail to meet. I believe that I am now enunciating a law of human nature, or human behavior. Let me restate it. The waving of a red cape before an enraged bull is trivial teasing compared with the effect on people whenever the desirability and attainability of an object vary inversely as the capacity or ability for attainment. Applied to mathematics, this means that the more queenly or radiant the handmaiden to the sciences is made to seem to the common man, and at the same time the more difficult or fruitless it is made for mastery or acquisition, the more the love and respect for mathematics are likely to turn into hate and disdain; and what might have been tender concern, into the will to annihilate.

Attacked on all fronts, secondary school mathematics has been rapidly retreating in not too orderly fashion. Occasionally we have witnessed courageous counter attacks but, generally, the defense not only lacked modern weapons, but suffered much from subversive influences, which, typical of subversive influences, often appeared in the friendliest guise. Very recently, however, it (secondary school

* Paper read at the section meeting of the Central Western Zone of the New York State Teachers Association in the Autumn of 1941.

mathematics) has acquired a new and powerful ally; a world catastrophe has come to its rescue; a national emergency is regaining for it much lost ground. Shall we let this blood and sweat opportunity turn our heads back to theories of divine right, special privileges, and the Maginot line security of a once again required subject?

The very same spokesmen for teachers who but a while ago were oozing sweet sentiments on relational thinking, and appeasing opponents at large by psychological arrangement of integrated subject matter are now saying, "We told you so; when boys and girls apply for defense jobs, the first question asked is, 'how much mathematics have you had?' We now dare educationists or the social studies group to dislodge us from our newly won position." And thus ladies and gentlemen, we are beginning to lay the foundation for what may prove to be total destruction of secondary school mathematics. A delusion of victory is more perilous than temporary defeat. *Mathematics for defense is but a temporary objective.* To overlook or ignore this fact might prove to be a costly, if not fatal, error.

When through fanatical fervor often accompanied by ignorance and occasionally admixed with malice, the "enemies" of secondary school mathematics attack its position in our schools, the loss to the general cause of education, though real, is only temporary. When through like fanatical fervor, a different brand of ignorance and a touch of group selfishness, the friends of secondary school mathematics defend its position in our schools, the consequence to the general cause of education may well prove to be more serious.

It would be rather sad if, among the large membership of our guild, there were not a few poets. I am sure that poet-mathematicians have as much right to be sentimental as other poets. Let our poets sing of Mathematics and Beauty; Mathematics and Citizenship; Mathematics and Art; Mathematics and Religion; Mathematics and Love; Mathematics and Social Consciousness; Mathematics and Humanity; Mathematics and Mercury; Mathematics and Breathing; Mathematics and Mathematics; God and Geometry, a la Plato; or God and Statistics, a la Jeans. *But let us beware of confusing poetry with objectives.*

Among professional people in general, and physicians in particular, chiropractors are held in none too deep veneration, not because they are a useless sub-class of the medical profession, but because they make such exaggerated and extravagant claims. Shall we, can we, rephrase our objectives and make them so specific that even the lowly educationist will know what we mean? Also, isn't it about time we made it quite clear to ourselves and others that secondary school mathematics is only a part of even an ideal secondary school curriculum? Furthermore, as a purely precautionary measure, let us be the first to admit that if every citizen of the U. S. A. were a mathematician, we might still not be an enlightened democracy and Utopia might still be far from rounding the corner.

It would be unfortunate, as it is practically inconceivable, if the profession lacked utilitarians—the bread-butterers of our craft, those who insist on practical use and application.

Say our friends, "Aren't we strengthening the position of mathematics by making courses in algebra and trigonometry so obviously practical?" Are we? When a course in mathematics becomes 99.44% applications, it will float by any other name. It may be trigonometry when we solve a triangle; it is an atrocity of city planning when we find the angles of intersection of the avenues forming a triangle. Worse than that, our most inventive text-book writers supply us with schemes for solving applied problems in mathematics without mathematics. A trivial illustration is that of the rectangular "meat chopper," in which by placing the proper ingredients in assigned rectangles and a counter-clockwise turn of the crank, the answer may be squeezed out into a vacant rectangular receptacle.

Another of our defenses is that mathematics is an unexcelled medium of postulational thinking which in literature and at our National Meetings became themes for discussion. Not so long ago, at one of our national meetings, for instance, a whole afternoon was devoted to topics subheaded "relational thinking." (I recall that at that meeting I asked a question which was not answered because the speakers thought I was being funny. I asked for a single illustration of thinking which was not relational thinking.) And I certainly need not remind you of the great accumulation of written and spoken material on "functional thinking." Right here I am going to take a stand which I know is not a popular one at the moment. I believe that we have given our opponents a solid basis for attacking secondary school mathematics when, in our arguments for retaining courses in secondary school mathematics, we so carelessly abolish the need for purely mathematical content. Let me explain this position by several illustrations: Suppose we defend geometry for its logical structure and then proceed to build a course which is replete with logical structure but lacking in geometry. Is it not clear that such procedure could be adopted by any subject matter area in the secondary school curriculum? If in the very demonstration of postulational thinking or logical structure or relational thinking or functional thinking, we employ material which is not geometrical in nature, are we not admitting that geometry, as such, is not essential to the development of that kind of thinking in our boys and girls? Take the indirect proof, for example. In a recently published textbook in geometry the bulk of illustrations is drawn from fields other than geometry; in other words, alibis accumulate while theorems in geometry decay. Why drag in geometry? I repeat that by building courses in geometry without geometry, we have made an implicit assertion that we can do all we claim to do with geometry, without geometry. I know some courageous theologians who suggest from time to time that a satisfying theology can and should be constructed employing postulational thinking. On a certain college campus, I know two people, one of whom, a historian by profession, flunked the only course in mathematics he ever attempted at college; the other, a mathematician, and a Phi Beta Kappa with all the trimmings. The former uses postulational thinking at its best; the latter avoids all varieties of thinking.

Many of the newer courses in mathematics are so "enriched" by non-mathematical material that it may soon require a special kind of bloodhound to trace the strictly mathematical scent. At its present rate of metamorphosis, geometry is rapidly becoming the fine embodiment of postulational, relational, functional, and as yet unnamed varieties of thinking nearly free from entanglements with purely geometric content. In fact, any day now it may become Course No. 3 in the Social Studies sequence.

Still another of our defenses is very familiar to all of us: "In mathematics a statement is either right or wrong." Educationists point to such a statement and say, "that plainly, is what's wrong with mathematics as a secondary school subject. It's unreal. Outside of mathematics, Q.E.D.'s are practically impossible, because no statement is wholly right or wholly wrong. Of what use is it to spend so much time fussing with neatly insulated, logical structures, when everywhere outside the mathematics classroom, whatever logical structures exist are all tangled up with unknown and sometimes unknowable variables!"

In almost every area there is lag between theory and practice. What I want to mention as one of the hurtful influences is more than this typical lag. It is a veritable chasm that exists between the vaporizations of our left-wing progressives at one extreme and the classroom practices of our reactionaries at the other. A wholly unprejudiced and fair minded educationist

might be expected to take into account the unique features of the two extremes, but is not such an attitude too much to expect? Consider an honest and sincere educationist who has an axe to grind. No selfish interest or vested rights or extreme prejudice are involved. Still, he does have a theory in which he would like to believe—let us say, that mathematics in the secondary schools for the greatest good of the greatest number should occupy a minor position. Such a person is not likely to destroy any evidence that supports his theory. Such a person is very likely to be influenced in a real way by some theory to which the bulk of good teachers of mathematics do not subscribe, and much practiec of which the bulk of good teachers of mathematics are ashamed.

I have mentioned the several groups of people among whom we find our friends, our enemies, and our friendly enemies. I have mentioned some actual and potential harm to mathematical education, and therefore all education, resulting from the very best intentions of our friends. On behalf of sane education I have suggested careful evaluations of carelessly constructed defenses. The latter are not peculiar to mathematics. Perhaps for some time now you have been expecting me to raise the question, "What, if anything, do we have in mathematics that they don't have in other subject matter areas?" In a recent picture, 'Caught in the Draft" in which Dorothy Lamour appears, a like question is asked about her—"What does she have that other women do not have?" The reply, somewhat audacious, is: "Nothing, only she groups it better." Perhaps that is true of mathematics. Perhaps mathematics is just better organized than the other branches of human learning. It may be that whatever it is Miss Lamour has is more amenable to good grouping, just as whatever it is mathematics has is more amenable to good organization.

Certainly many of us are convinced that in the most effective scheme of education in a Democracy there can be no substitute for mathematics—that is, for appropriately rigorous, well organized, well-taught mathematical content. To defend any other kind is to hurt the good cause. It is as futile as it is unnecessary to defend courses in mathematics by eliminating mathematics from these courses.

And now with due apologies to you, I am going to descend to a bit of preaching.

Our real long range job is not so much convincing educationists, or administrators, or even mathematicans. It is convincing—in the only genuine way there is—the *learners* of mathematics. The greater the number of today's boys and girls learning mathematics, the fewer tomorrow's adults likely to have either cause or inclination to attack it. As teachers, let us reduce to a minimum our self-indulgence and the consequent need for shifting the blame on the "system," on imposed examinations, on administrative difficulties; in short, on anything and everything *save* our own inertia, or indifference, or the depth and narrowness of our ruts.

Finally, many of our best friends say that mathematics needs no defense. Certainly, it doesn't. What branch of human learning needs defense! Neither "our friends" nor "our enemies" can change the essentially quantitative aspect of the world we live in. Our concern is not the welfare or well-being of mathematics, nor even the welfare or well-being of teachers of mathematics. It is, as it must be, the welfare and well-being of boys and girls who can be made better human beings by fertile schooling in genuine mathematics.

Distribution of Content in Twenty-three Ninth-grade Mathematics Textbooks for Non-academic Pupils

By MRS. FAITH F. NOVINGER
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TO WHAT extent do textbook writers agree on the content to be included in the ninth-grade mathematics textbooks for the non-academic pupil? What are the tendencies with respect to emphasis on subject matter in these books? To answer these questions the writer analyzed the distribution of content according to subject matter in twenty-three ninth-grade mathematics textbooks, published from 1934 to 1940, which are designated as suitable for the non-academic group. Four of the texts studied are business arithmetics. The remainder of the books deal with "social mathematics," which means "practical applications" of mathematics. Three of the textbooks represent the third volume in a series of three books. The rest are "terminal" texts, since no others have been written.

As a means of securing a basis for comparison, a master list of all categories in the field of mathematics was compiled from the tables of contents. The amount of space devoted to each category was determined by counting the pages given to that category in each book. The major categories of subject matter found in these ninth-grade mathematics books include *arithmetic, geometry, algebra, social uses* and *consumer training.* Not all of these categories are included by all the authors. Arithmetic is the sole category on which all of them agree as essential to ninth-grade mathematics. Commercial arithmetic ranks second; geometry is third in rank; and algebra fourth, in their opinion. The relative emphasis on these categories is shown in Table I and Figure 1.

The categories in the field of mathematics found to be most important as shown by page space are: (1) operations of arithmetic; (2) commercial arithmetic; (3) geometry; (4) algebra; (5) social uses; (6) consumer training. Less prominent divisions include graphs, industrial arithmetic, trigonometry, tables, tests and the history of mathematics. Table I lists these topics in the order of their frequency.

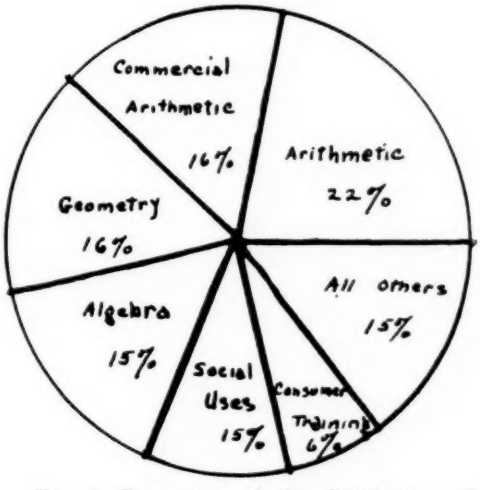

FIG. 1. Percentage of the distribution of divisions in twenty-three ninth-grade mathematics textbooks for non-academic pupils.

TABLE I

Content of Twenty-Three Ninth-Grade Mathematics Textbooks for Non-Academic Pupils

Division of Content	No. of Pages	Percentage Distribution
Arithmetic	1914	22
Commercial Arithmetic	1569½	16
Geometry	1512½	16
Algebra	1437½	15
Social Uses	906½	10
Consumer Training	562½	6
Graphs	511	5
Industrial Arithmetic	388	4
Trigonometry	225	2
Tables	213	2
History of Mathematics	60	1
Miscellaneous	134	1
Total	9433½	100

The operations of arithmetic rank first, with 1914 pages, or twenty-two per cent. Commercial arithmetic, often the substitute for ninth-grade algebra, ranks second, with 1569½ pages, or *sixteen* per cent. Geometry ranks third, with 1512½ pages, or *sixteen* per cent. Algebra, the traditional ninth-grade subject, ranks fourth, with 1437½ pages, or *fifteen* per cent. Social uses rank fifth, with 906½ pages, or *ten* per cent. Consumer training ranks sixth, with 562½ pages, or *six* per cent. Graphs rank seventh, with 511 pages, or *five* per cent. Industrial arithmetic ranks eighth, with 388 pages, or *four* per cent. Trigonometry, Tables, and History of Mathematics rank ninth, tenth, and eleventh with 225, 213, and 60 pages, which is *two* per cent, *two* per cent, and *one* per cent respectively. Certain miscellaneous articles not readily classifiable made up 134 pages, or *one* per cent, of the content.

Relative importance of certain topics in general mathematics. Another way of determining the importance of certain categories as shown in the twenty-three ninth-grade mathematics textbooks is to see how many make provisions for a given category.

As shown by Table II, provisions for the operations of arithmetic and tables are made by all twenty-three textbooks. Graphs are found in twenty-two textbooks; geometry, consumer training, and commercial training are included by

TABLE II

Importance of Certain Topics Determined by Frequency of Appearance in Twenty-Three Mathematics Textbooks for Non-Academic Pupils

Topic	Number texts making provision	Number texts omitting provisions
Operations in arithmetic	23	0
Tables	23	0
Graphs	22	1
Geometry	21	2
Consumer Training	21	2
Commercial Training	21	2
Social Uses	20	3
Algebra	16	7
Trigonometry	15	8
Industrial	11	12
History of Mathematics	9	14

TABLE III

The Range of Certain Categories in Mathematics in Twenty-Three Textbooks for Non-Academic Ninth-Grade Mathematics Pupils

	Operations of Arith.	Commercial Arith.	Geometry	Algebra	Social Uses	Consumer Training	Tables
1.	219	293½	132½	207	122	107	35½
2.	168	187½	126½	205	83½	70	29½
3.	158	182	121	143	72	60	17½
4.	135	168	120½	129	62½	58½	14
5.	131½	118	113½	123	60	54	14
6.	125	106	102	117	55	34	12
7.	108½	104	96	86	51	28	11
8.	106	80½	86½	81	50	28	9½
9.	97	74	75	81	49	17	9
10.	81	68	70	75	42	16½	8½
11.	77½	38	67½	57½	38½	11½	7
12.	77	32	65	41	36	11	6
13.	65½	24½	65	39	32	10	5½
14.	53	24	59	36	31½	9½	5
15.	52	17½	50	12	30	9	5
16.	49	15	49½	4	26	8½	5
17.	48	10	46½	0	25½	8	4
18.	40	10	23	0	19	7	4
19.	37	8	18	0	17	5½	3
20.	34½	7	17	0	4	5	3
21.	28	2	9	0	0	4	2
22.	20½	0	0	0	0	0	2
23.	4	0	0	0	0	0	1
	1914	1569½	1512½	1437½	906½	562	213

twenty-one; social uses are included by twenty; algebra is included by only sixteen; trigonometry is included by fifteen; industrial mathematics is included by eleven; and the history of mathematics is included by nine.

The authors are in agreement on *arithmetic* as a subject for the non-academic ninth-grade mathematics pupils. They are also in agreement on the topic of tables. It is interesting to note the tendency to include *consumer education* equally as often as commercial education. This seems to reflect a tendency in this country to be concerned with consumer problems. There is a a great diversity of opinion concerning algebra. Seven of the texts omit algebra altogether. However, the sixteen authors who do include algebra believe it important enough to devote 1437½ pages to it.

The range of subject matter among the various divisions will be presented inTable III at this time, because this information is needed for an understanding of the data cited later in this discussion.

By page content, *arithmetic* is the outstanding subject for non-academic ninth-grade mathematics pupils. The content of arithmetic in the various books ranges from four to 219 pages, as shown by Table III. The median number of pages in arithmetic is 77 pages. Eight of the books for non-academic ninth-grade mathematics pupils contain more than 100 pages of arithmetic, fifteen contain less than 100 pages.

The reader will remember that Table I shows that 1569½ pages, (or 16% of the content) deal with *commercial arithmetic.* One of the first breaks with the traditional college preparatory course of ninth-grade algebra was commercial arithmetic. Many non-academic pupils have been in commercial arithmetic classes. Twenty-one books include commercial arithmetic. One book has 293½ pages of commercial arithmetic work. Six of the books tend to have about 150 pages. The remainder center around 40 pages. It would seem that some of the books have a definite commercial objective, while the remainder of the books treat the commercial work as part of the general mathematics training.

As shown in Table III, in the sixteen books containing *algebra,* the range is from four to 207 pages. Six books have more than 100 pages, two of the six have more than 200 pages, ten books have less than 100 pages. It would seem that *there is no guiding principle, nor unanimity of opinion among the authors in this connection.*

Geometry comprises 16% of the content of the twenty-three mathematics books for non-academic ninth-grade mathematics pupils. The range found in geometry in 21 of these textbooks is from nine to 132½ pages, the median being 67½ pages. Six books include more than 100 pages, fifteen have less than 100 pages devoted to geometry. The books agree as to the amount of content in geometry to a greater degree than they do in either arithmetic or algebra.

The topic of *social uses* is included by twenty authors. The content ranges from four pages to 122 pages. The median is 40¼ pages. One book contains 122 pages; three books have 83½, 72, 62½, and 60 pages; five center around 50 pages; seven center around 30 pages; two center around 20 pages; and one has four pages.

Consumer training is included by twenty-one authors. The content ranges from four pages to 107 pages. The median is 11½ pages. The highest number of pages is 107; four center around 60 pages; three center around 30 pages; five center around 15 pages; the remaining eight center around 8 pages.

The topic of *tables* is included more fully, because, as shown by Table III, there is entire agreement among the authors about offering tables. It was interesting to find that some books confine the tables to the appendix. This would probably account for the seven books which have a range of from one to four pages devoted to this topic. In the other books the tables are scattered throughout the books. The range is from one to 35½ pages. Seven

have from 11 to 35½ pages, and nine have from five to 9½ pages.

Findings. I have discussed the extent to which the authors of twenty-three ninth-grade mathematics textbooks agree on the content for non-academic pupils. Some of the major findings of the study are as follows:

1. The major categories of subject matter found in these ninth-grade mathematics books include *arithmetic, commercial arithmetic, algebra, social uses* and *consumer training.*

2. Not all categories are included by all authors. With respect to the selection of content for ninth-grade mathematics, textbooks, *arithmetic* and *tables* are the only topics on which the authors are in agreement.

3. *Arithmetic* is the category which receives the greatest emphasis as shown by page space. Twenty-two per cent of the entire page space is devoted to this topic. However, the authors show little agreement concerning the range of arithmetic, since it varies from four to two hundred nineteen pages in the 23 books.

4. There is great diversity of opinion concerning *algebra.* Seven of the texts omit it altogether. However, the sixteen authors who do include algebra believe it to be important enough to devote fifteen per cent of the entire page content of all 23 books to it. There is also a wide range in this topic, for it varies from four pages to 207 pages.

5. In *geometry* there is closer agreement among the authors concerning the amount to be included than in either arithmetic or algebra. The general tendency is to devote about one hundred pages to this topic.[1]

[1] In order to see if these tendencies are continuing in books published in 1941, the present writer checked four of the recent textbooks and found the distribution of subject matter to be about the same. However, the subject of *science* was included as a new category in two texts, with 10 and 22½ pages devoted to it, respectively.

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Why Teach Mathematics?*

By LUCIEN B. KINNEY
Stanford University, California

A FAVORITE argument in educational circles, in times not long passed, concerned the relative importance of subject-matter, as compared to methods, in the preparation of the teacher. Stated positively, the subject for debate might be stated in the cliché, "If you know your subject you can teach it." Today we recognize that both are necessary, and neither is useful without the other. Yet even together they are not sufficient without a third component—an appreciation of the purposes of education here and now.

No one will deny the importance to the mathematics teacher of his scholarly knowledge of the field. Without it he cannot see the significance of mathematics in solving the problems of society, nor reveal it to his pupils. How can he show them the road if he himself cannot perceive the goal?

Knowing his field and understanding its potential contributions to the purposes of education, but without command of the educational procedures that will make his field effective in achieving them, the teacher is still inadequately prepared. The planning and directing of effective learning experiences requires an understanding of human behavior, its control, prediction, and modification. Such an understanding may be acquired as part of professional education, or as the result of experience. If it is lacking, the teacher is in danger of becoming a starry-eyed dreamer, who can discuss educational problems in futile verbalisms, but whose pupils lead a butterfly existence among whatever activities their fancies may elect.

But what of the teacher with a scholarly knowledge of his field, who has acquired through experience or study those techniques of classroom organization and procedure that go to make up efficient methodology, yet who does not comprehend the role that mathematics teaching should play in the total program of education? Someone has defined a fanatic as one who, having lost sight of his goal, redoubles his efforts. If we accept this definition, we see that such a teacher is in danger of becoming a fanatic. Losing sight of what the schools are for, he comes to teach his subject as an end in itself. All of us are devoted to the logical organization of subject-matter. The great fields of human knowledge such as mathematics and science are peculiarly susceptible to such organization. As system and logic assume greater emphasis, relationships to the purposes of the school may be overlooked. This is not a circumstance peculiar to mathematics teachers, though the acknowledged importance of their field and its logical nature makes them peculiarly susceptible to it. Probably most of us are fanatic at one time or another, and doubtless some of the most pronounced fanatics are in institutions of higher learning, even the teachers colleges themselves.

The best safeguard against such fanaticism lies in never losing sight of the question we have raised for this evening: Why teach mathematics? An answer to this question, it seems to me, must be based on two assumptions, both of which appear eminently reasonable. The first is that each field must contribute directly to the general aims of education. Some statement of the purposes of education is therefore in order at the outset. The second is that any field, to deserve a place in the curriculum, must have a unique role to play in achieving these purposes. Whatever contributions are claimed for such a field must be established by something more than theoretical suppositions. The outcomes must be verifiable through observed modifica-

*A paper read at the meeting of the North San Joaquin Valley Teachers Association at Stockton, California, November 14, 1941.

tions in pupil behavior as the result of experiences directed by the school.

These assumptions, then, lead us first to the question: What are the schools for? In every country, and in every period of history, whatever program of education is set up has been designed to enable people to play their part in the community life and assume their share of its work. From the emphasis that is placed on various aspects of this purpose during any given period we obtain some indication of the most pressing needs of the culture and the times. Today, when democracy, which has more or less been taken for granted in the past, is threatened both from within and without, it is only natural that statements of educational purposes are quite preoccupied with problems of education for democracy. What is democracy? What kind of people are necessary to make it work? What kind of schools do we need in a democracy? Such questions as these were uppermost in the thoughts of the Educational Policies Commission, when they defined[1] the areas in which educational objectives are to be realized in the modern American society. The direction of thinking in the Report[2] of the Committee on the Function of Mathematics in General Education is toward the achievement of the same general purposes. We find, for example, that "the major values to be fostered are those associated with democracy as a way of life. . . . Acting on this belief, the Committee presents its view as to the meaning of democracy and indicates some implications both for the school in general and for the mathematics program."[3]

The concept of democracy, as defined in these and other reports, goes beyond its political connotations. It is thought of as a way of life, which we believe best in the long run. Members of a democracy are something more than an association of individuals, in a laissez faire sense. They share in common purposes, which each member accepts as his own. They share in planning and executing the means to accomplish these purposes. The effectiveness of democratic procedures depends on the individual effectiveness of the members of the democracy. Our next question, then, may well be: What can we do in the mathematics class to prepare our pupils to play their part in a democratic society?

The unique contribution of mathematics, of course, is the quantitative mode of thought, which is illustrated by the mathematical approach to a problem. "The mathematics teacher makes his special contribution whenever quantitative data and relationships, or the facts of space and form are encountered."[4] The Report of the Educational Policies Commission[5] defines four areas, in which objectives of education are to be set up. It will be interesting to examine these four areas, and note what contributions mathematics can make toward directing growth in each area. The four areas of the Commission are as follows:

Self Realization, which is centered on the personal development, growth, and learning of the individual;

Human Relationships, concerned with the relationships of the individual with his home and family, neighbors, and community;

Economic Efficiency, which includes the education of the individual both as a producer and consumer, as well as an investor;

Civic Responsibility, involving his relationships with his governments, and with the people of other nations.

Merely a casual overview of these areas is sufficient to reveal that mathematics has much to contribute in each. Let us examine them each more closely.

[1] Educational Policies Commission, *The Purposes of Education in American Democracy*, Washington, D. C., N. E. A., 1938.

[2] Commission for Secondary School Curriculum, *Mathematics in General Education*, New York, Appleton-Century, 1940.

[3] Ibid., pp. 24, 25.

[4] Ibid., p. 45.

[5] Educational Policies Commission, *The Purposes of Education in American Democracy*, pp. 45–47.

Self Realization. Here the point of view of democracy is that the potentialities of the individual are to be emphasized. While the state exists for the welfare of the individual, the success of democratic procedures depends on the ability of the individual to play his part. In the last analysis, what can we do in the mathematics class to contribute to the realization of the individual's potentialities?

1. We can provide the citizen of tomorrow with those mathematical procedures that are necessary for him to deal masterfully with his personal affairs and his share of the community enterprise. The ineptitude of the typical adult when confronted with a problem involving fractions, percentage, or often simple computations with whole numbers, defines the outcome to some extent. The level of mathematical literacy required for these purposes is continually rising, and the attention focussed on it recently by the defense emergency will in all probability be something permanent.

2. We can direct the growth of the pupil's facility in communicating and interpreting mathematical ideas. A public that is intelligent on common problems requires the use of charts, graphs, tables, and formulas, not to mention percentage and numbers. Recent reports of the National Resources Commission, and various agencies in the field of social security, illustrate the extent to which public reports are coming to expect a considerable degree of familiarity with these means of communicating quantitative ideas. Their use in newspapers and periodicals is steadily increasing.

Effectiveness in dealing with social and economic issues, moreover, requires that the individual must be able to utilize the concepts that are basic in the quantitative mode of thinking—functional relationships, approximate nature of measurement, those related to statistics, geometry, and trigonometry, as well as the use of symbols. All these are essential in dealing with the problems of modern society.

3. We can direct the growth and development of desirable and useful personalities. Buswell, in the Sixteenth Yearbook of the National Council of Teachers of Mathematics,[6] has pointed out two implications for mathematics in the field of personality. The first he refers to as the negative. This is illustrated by the feeling of inadequacy revealed by many adults in the presence of a mathematical situation, which in many cases amounts to a strongly emotional reaction. Many students in college experience an emotional block of this sort whenever a topic is presented that uses the mathematical approach. This reaction can usually be attributed to unfortunate experiences in the previous courses in mathematics. We can never afford to overlook the fact that the pupil learns by wholes—he responds to the whole situation, and the whole pupil is involved in the learning process. It is unprofitable for him to acquire a skill if at the same time he is acquiring an unfavorable attitude toward the field.

The second implication is positive. It is the possibility of providing experiences that develop a feeling of adequacy in meeting the real situations of life. The feeling of competence that comes from mastery over them is one of the most important personality needs of every pupil. The mastery of mathematical relationships is essential in making adjustments to many of these situations. Numerous problems of this sort occur in the life of the pupil. Other frustrating experiences are reserved for adult life, although the pupil may be led to understand and appreciate them. In either case the personality development that comes from experience in overcoming obstacles is an important outcome from the mathematics class.

Human Relationships. The outcomes of education that go to make up team-

[6] Guy T. Buswell, "The Function of Subject-Matter in Relation to Personality" Chapter II in *Arithmetic in General Education*, 16th Yearbook, NCTM, Bureau of Publications, Teachers College, 1941.

work efficiency are included in the area of Human Relationships. A democratic society requires that each individual develop effectiveness in group effort. What can we do in the mathematics class to increase his ability and interest in the use of democratic procedures?

1. We can provide him with effective instruments for group activity. Many of the concepts of mathematics were historically necessary for cooperation among members of society. Judd, for example, in his Psychology of Social Institutions, points out the importance to society of the psychology of precision. How could we standardize the parts of a machine, operate a railroad or retail store, or wage a war without measurement? There is also the psychology of punctuality—all of us know from experience how difficult it is to cooperate with the individual who never knows what time it is. The individual who deals quantitatively with his time, as well as with his other resources, is not only more efficient as an individual, but as a cooperating member of a group.

2. Going beyond these concepts, it has been experimentally established that mathematics provides an effective medium for developing those attitudes and abilities needed for reflective thinking. In a totalitarian state this would not be a desirable outcome from the educational program. There the thinking of the individual is done for him, and his duty is to accept and comply. To the degree that every individual participates in determining the goal of public effort, as well as in planning and carrying out the activities necessary for achieving them, we have a democracy. It follows, then, that democratic procedures are efficient only if the individuals are capable of critical, reflective thinking.

While not all problems are quantitative in nature, and therefore susceptible to mathematical treatment, none the less, mathematics has a general contribution to make in throwing the processes of problem solving into clear relief. This aspect of mathematics is treated at some length in the Report of the Committee on the Committee on the Function of Mathematics in General Education, as well as in the Thirteenth Yearbook of the NCTM.

Economic Efficiency. The area of Economic Efficiency includes two general sub-areas: Vocational Effectiveness, and Consumer Effectiveness. Does the mathematics teacher have a contribution to make in these sub-areas? The answers here are rather obvious. There is little need to dwell on the vocational value of mathematics, particularly if we include here the college preparatory function of mathematics. Historically speaking, it has continually increased in its economic importance. Introduced originally for its necessity for surveying, navigation, and business, its direct vocational values today are multiplied many times. The problem has always been for the school to keep pace with industry in recognizing the applications of mathematics in technical and occupational problems.

The importance of the consumer aspect of the economic area has not received so much recognition as the producer aspect. It is true that 150 years ago, when arithmetic was being taught at Harvard, Adam Smith was calling attention to consumption as the chief end and purpose of production, and pleading for recognition of the consumer's problems. Since that time the problems of production have, as a whole, been solved. The problem of getting the products of the economic system to its individual members in the amounts needed by each still remains practically untouched. It was probably excusable, in the past, for us to assume that if the problems of production were solved, the evils due to scarcity would disappear. The paradox of want due to overproduction may well have been somewhat beyond the realm of human prediction. Yet today there is need for us to recognize the responsibility of the school for preparing an intelligent consumer, whose tastes and styles will intelligently direct the activities of industry, whose purchasing power and investments

can keep the wheels turning, and whose clear understanding of the responsibility and function of each unit in the economic system will make it possible for him to hold it to strict accountability for its obligations to the consumer.

The possibilitites for consumer education through mathematics are many and far-reaching. There is no better way of bringing about an understanding of the consumer as an economic entity, with his potential control of production, and his responsibilities for directing industry. Most of the problems of the consumer are quantitative in their nature. Even those relating to leisure time activities and intellectual advancement require the solution of sordid problems of family finance before they become real. It is true that we have made considerable progress in this area. Surveys of school practices indicate that, as compared to other fields, mathematics classes carry a large share of the consumer education that is provided. As yet, however, we must realize that we have hardly scratched the surface, or explored the possibilities.

Civic Responsibility. The area of Civic Responsibility is largely an area of attitudes. Those in question are the ones that comprise the team-spirit of democratic society. Among the attitudes mentioned as important, for example, are social sensitivity, aesthetic appreciation, and cooperation. Each classroom has the responsibility for directing growth in these attitudes, mathematics along with the rest. Yet we may well ask, does the mathematics class offer unique possibilities for developing these attitudes, such as are not afforded by other fields?

If we agree that a sense of group responsibility for economic security of the individual is an aspect of social sensitivity, the writer can testify that it does. He has observed a class in the process of acquiring that attitude during a mathematical discussion of the problems of a family in the median income group, as it attacked the question of providing protection against economic hazards. The quantiative approach provided by mathematics removed the question from the realm of politics and prejudice, and led inescapably to the conclusion that such a family was not able, through its individual efforts, to provide its own protection through savings. The implications for group responsibility were clear to every member of the class, and came as his own solution of the problem.

What about aesthetic appreciations? While there is little conclusive evidence in this area, we are certain that such concepts as rhythm, proportion, balance, symmetry, and so on, which are essentially mathematical, are basic to certain areas of appreciation. Understanding of these concepts, while not sufficient to guarantee appreciation, is certainly a necessary condition. That many children and adults recognize and appreciate these features of their environment, and yet have a distaste for mathematics, is merely evidence that these appreciations are being developed elsewhere, and that many of us in mathematics are missing an opportunity.

We have noted previously that efficiency of the individual members of the group in the procedures for group activity is not a guarantee of group efficiency. There must be an attitude on the part of each member of the group that will lead to a desire for group goals and participation in democratic procedures for attaining. An attitude of cooperation exists when each member feels a personal share in these goals, and a personal responsibility for seeing that they are attained. The strong morale of a well-balanced team, or of a trained army, is due to this feeling. Such attitudes are the result rather of how the purposes of the class are achieved, than of what subject-matter is taught. The identical mathematical topics might be learned by pupils in a totalitarian state, a democracy, and a highly individualistic society. Yet whether there is education for submissiveness, for cooperation, or for competition would depend on the classroom procedures and social atmosphere. Our teaching tech-

niques, in brief, must be chosen in the light of pupil needs, and the necessity for developing techniques and attitudes for cooperative living. The procedures that have been devised recently for evaluating classroom outcomes such as attitudes and appreciations should be utilized to determine the effectiveness of our classroom activities in achieving them.

Returning to our question for the evening: Why teach mathematics? We can now answer it positively and briefly. We teach mathematics because it is an effective means for achieving the purposes of general education. It has a contribution to make to each of the areas in which objectives are set up that can be achieved by no other field of subject-matter. It is true that other subjects have their contribution to make, and that cooperation is desirable, and in fact necessary, among all the fields. Yet the outcomes from mathematics are so essential to every member of a democracy that no pupil should be deprived of the opportunity to benefit from the experiences it can provide.

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Minutes of the Eighth Winter Meeting of the National Council of Teachers of Mathematics

By EDWIN W. SCHREIBER, *Secretary*
Macomb, Illinois

THE EIGHTH WINTER Meeting of the National Council was held from December 31, 1941 to January 1, 1942, at Lehigh University, Bethlehem, Pennsylvania. About 200 were in attendance at the various section meetings of the Council and 170 registered.

Six sessions were held on Wednesday:

General Session at 10:00 A.M., Christmas Saucon Hall, Lehigh University.

Multi-Sensory Aids demonstrations at 1:00 P.M., Chemistry Building.

Junior High School Section at 2:30 P.M., Christmas Saucon Hall.

Senior High School section at 2:30 P.M., Christmas Saucon Hall.

Teacher Training Section at 2:30 P.M., Christmas Saucon Hall.

Joint Dinner with the American Mathematical Society and the Mathematical Association of America, Hotel Bethlehem, 8:00 P.M.

Five sessions were held on Thursday:

General Session at 10:00 A.M., Christmas Saucon Hall.

Multi-Sensory Aids demonstrations at 1:00 P.M., Chemistry Building.

Senior High School Section at 2:30 P.M., Christmas Saucon Hall.

Teacher Training Section at 2:30 P.M., Christmas Saucon Hall.

Multi-Sensory Aids at 2:30 P.M., Christmas Saucon Hall.

The complete program of this meeting is published in THE MATHEMATICS TEACHER for November 1941, pages 289–290. The National Council is very much indebted to Dean Tomlinson Fort and his staff of Lehigh University for the fine accommodations given to the Council for holding its meetings. The following committees are also to be congratulated for the services rendered:

The Exhibits Committee, Housing Committee, Publicity Committee, and the Local Arrangements Committee.

The Board of Directors had a meeting beginning at 1:00 P.M., Tuesday, December 30 at the Hotel Bethlehem. Two adjourned sessions were also held—one at 7:30 P. M., December 30 and at 5:00 P.M., December 31.

PRESENT FROM THE BOARD: Mary A. Potter, F. L. Wren, R. L. Morton, Edwin W. Schreiber, W. D. Reeve, W. S. Schlauch, Virgil S. Mallory, H. W. Charlesworth, L. H. Whitcraft.

ABSENT FROM THE BOARD: Vera Sanford, A. Brown Miller, Dorothy Wheeler, C. M. Austin, Hildegarde Beck, Kate Bell, J. O. Hassler.

OTHERS PRESENT: Martha Hildebrandt, Kenneth Brown, William Betz, E. H. C. Hildebrandt.

President, Mary A. Potter of Racine, Wisconsin, called the meeting to order at 1:10 P.M. Mr. Kenneth Brown, Chairman of State Representatives, made a brief oral report on membership of the National Council. He stated that the present membership in round numbers is 5,800. Some states have lost members, while others have gained. No official action was taken on Mr. Brown's report.

It was moved by Mr. Wren, seconded by Mr. Mallory, and carried that we dispense with the reading of the minutes of the last meeting of the Board of Directors since it was published in the May 1941 issue of THE MATHEMATICS TEACHER.

It was moved by Mr. Schlauch, seconded by Mr. Charlesworth, and carried that the price of the 17th Year Book be not less than $2.00, and as much more than $2.00 as present business conditions warrant so that we make the same percentage profit as we have in past Year Books.

It was moved by Mr. Morton, seconded

by Mr. Wren, and carried that Editor Reeve investigate present binding costs, and that he be authorized to have past Year Books bound if he considers it to be a good business investment.

It was moved by Mr. Morton, seconded by Mr. Whitcraft, and carried that on and after May 1, 1941, all previous Year Books be sold at $1.75 per copy.

It was moved by Mr. Mallory, seconded by Mr. Reeve, and carried that $50.00 be allowed the 17th Year Book Committee.

It was moved by Mr. Morton, seconded by Mr. Wren, and carried that Professor Reeve be responsible for making a detailed report on all Year Books—one to sixteen—and submit same report to the Board of Directors within a reasonable length of time.

Professor E. H. C. Hildebrandt made a very interesting and detailed report on the progress of the Multi-Sensory Aids Committee and the forthcoming Eighteenth Year Book.

It was moved by Mr. Schreiber, seconded by Mr. Mallory, and carried that Professor Hildebrandt investigate the possibility of assembling a kit of samples of Multi-Sensory Aids and also to determine the approximate price of such a kit.

President Mary A. Potter made her biannual report as President of the National Council, which was accepted with thanks and is to be placed in the President's Book.

A brief report on the Dictionary Committee was made by Mr. Wren.

A written report of the Radio Committee prepared by Mr. Miller was read by Secretary Schreiber, accepted and placed on file.

It was moved by Mr. Mallory, seconded by Mr. Charlesworth, and carried that Professor Reeve be authorized to publish a monograph entitled, "The Story of Measurement."

It was moved by Mr. Mallory, seconded by Mr. Wren, and carried that the matter of publicity for the National Council be placed in the hands of Mr. Brown, Chairman of State Representatives.

It was moved by Mr. Mallory, seconded by Mr. Wren, and carried that we continue the services of Mr. Brown as Chairman of the State Representatives at the present salary.

It was moved by Mr. Reeve, seconded by Mr. Charlesworth, and carried that a committee be appointed to stimulate the activities of affiliated organizations by giving them some projects to work out.

It was moved by Mr. Whitcraft, seconded by Mr. Charlesworth, and carried that during the duration of the emergency, we eliminate the winter meeting.

It was moved by Mr. Schlauch, seconded by Mr. Mallory, and carried that in future meetings no money be allowed for speakers.

It was moved by Mr. Reeve, seconded by Mr. Schreiber, and carried that during the emergency the Official Ballot for determining officers of the National Council be printed in THE MATHEMATICS TEACHER, and not be mailed out on Government Post Cards as has been the practice the past four years.

Mr. Charlesworth presented the report of the Committee on Reorganization and after considerable discussion it was moved by Mr. Schreiber, seconded by Mr. Reeve, and carried that the report of the Committee be accepted with thanks and the committee continued until the next annual meeting of the National Council in 1943.

The following amended budget was established for the year 1942–43:

Annual Meeting	$ 440.00
Summer Meeting	200.00
State Representative Committee	250.00
Salary of Chairman of State Representatives	600.00
Defense Committee	25.00
President's Office	250.00
Secretary-Treasurer's Office	515.00
Multi-Sensory Aids Committee	400.00
Seventeenth Year Book Committee	50.00
Total	$2730.00

Attendance at Bethlehem Meeting, National Council of Teachers of Mathematics

December 31, 1941–January 1, 1942

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Wirth, Richard
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Black, Amos

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STATES	MEMBERS	NON-M.	TOTAL	STATES	MEMBERS	NON-M.	TOTAL
Cal.	1	0	1	N. Car.	1	0	1
Col.	1	0	1	Ohio	4	2	6
Conn.	2	0	2	Ore.	1	0	1
Del.	1	1	2	Penn.	43	40	83
D.C.	4	0	4	R. I.	1	2	3
Ill.	6	3	9	S. Car.	1	0	1
Ind.	1	1	2	Tenn.	1	0	1
La.	1	0	1	Vt.	1	0	1
Md.	2	1	3	Va.	1	1	2
Me.	0	1	1	W. Va.	2	0	2
Mass.	0	1	1	Wis.	1	1	2
Mich.	3	0	3	Wyo.	1	0	1
Mo.	1	0	1				
N.J.	10	9	19	Total	103	67	170
N.Y.	11	4	15				

A Reply to Mr. Nygaard

By NORMAN N. ROYALL, JR.
Winthrop College, Rock Hill, South Carolina

I HAVE read, with ever increasing incredulity, an article in the October (1941) issue of THE MATHEMATICS TEACHER entitled "A Functional Revision of Plane Geometry" by P. H. Nygaard. Mr. Nygaard's article is such a glaring example of the type of discussion which finds its way into print today to the mortal harm of sound instruction in mathematics that I can not let it pass unchallenged. We have here at Winthrop a student chapter of the National Council of Teachers of Mathematics into the hands of whose members there comes each month during the school year a copy of THE MATHEMATICS TEACHER. Since I am the faculty sponsor of this group I can for them correct the errors in Mr. Nygaard's essay. The effect of this correction is, however, limited by the range of my voice; therefore, I hope that my reply may have full publicity in THE MATHEMATICS TEACHER to the end that I may reach the audience afforded to Mr. Nygaard.

Mr. Nygaard begins by taking to task the inept geometry teachers for their boast that geometry is over two thousand years old, that the science is much as Euclid left it, and that revision is unnecessary "because mathematical truths never change." I quote extensively from Mr. Nygaard:

". . . To illustrate their view, they (the geometry teachers) argue along the line that if the sum of the angles of a triangle was 180° in Euclid's day, the sum will be 180° today and forever. This infallibility, or permanence, they insist is one of the inherent characteristics of mathematics, and is sadly lacking in other subjects, especially the sciences.

"Such boastfulness is, however, largely misplaced. Mathematics teachers should be thoroughly ashamed of a subject as decrepit as plane geometry. Since Euclid there have occurred countless developments which should have logical repercussions in the teaching of geometry.

"One of these developments has been the extensive replacement of deductive reasoning by inductive reasoning. Aristotle and Euclid proved proved specific truths by showing that they were the consequences of God-given, obvious, general laws. From the time of Roger Bacon and Galileo this procedure has been largely reversed—general laws are derived through a process of classifying numerous specific facts. Nor has this upheaval been confined to science alone; the very theorem just mentioned regarding the sum of the angles of a triangle has been questioned by modern mathematicians. Geometry can ill afford to depend entirely on deductive reasoning when nearly all science and philosophy rely on inductive pragmatism."

I pass over the thought displayed in the second paragraph quoted wherein Mr. Nygaard assumes that developments in the technique of teaching geometry can affect the subject of geometry itself in an essential manner. This is one of the most common—and vicious—fallacies in much of modern educational discussion: that a psychology of methodology can of itself determine content.

Let us come to the main point. The paragraphs quoted show an entirely erroneous conception of the nature of "mathematical truths." The proper word to use in connection with pure mathematics is not "truth" but "validity." Even poor, outdated Euclid seems to have had somewhat more of a glimmer of this idea than the author of the above paragraphs, for the famous parallel postulate of Euclid on which the theorem concerning the sum of the angles of a plane triangle is based was considered far from "obvious" and "God-given" by the Greeks. Serious attempts were made to deduce this postulate from the others.

And in just what precise sense has this "very theorem just mentioned concerning the sum of the angles of a triangle . . . been questioned by modern mathematicians"?

Why, simply this: by *assuming postulates*

different from Euclid's parallel postulate Lobachevsky and others developed an equally consistent and "valid" geometry in which this theorem does not have the form it has in Euclid. Does this mean that Euclid's theorem is no longer "true"? Certainly not. What it means is that we have found out something important about the nature of mathematics. Both are valid on the basis of the particular set of assumptions from which each was deduced. And which set of assumptions is true? No—"modern mathematician" attempts to say. This is not the province of mathematics, but of physics, or epistemology perhaps. It is the business of mathematics, as pure mathematics, to be concerned with "validity."

It should perhaps be pointed out that things *have* happened to pure geometry since Euclid's day. For example the great German mathematician Hilbert has put the deductive structure of geometry on a more rigorous basis. He has done this by improving and extending, not abandoning, the deductive method. The debt of Hilbert and Lobachevsky is to their great teacher, Euclid—not to Roger Bacon. You can be safely informed that what these modern mathematicians have done has distinctly *not* been the result of running around with a protractor measuring the angles of hundreds of triangles in an attempt to find out "the real life situation."

It will perhaps be objected that maybe I have just misunderstood Mr. Nygaard and am reading too much into his statements, but I think not. I quote again:

"The writer has made some effort to determine what material, usually included in plane geometry, is of little or no value in later mathematics or science courses. . . .

"Other time consuming items can be eliminated by applying the criterion that it is unwise to thresh over in minute detail ideas which the students have studied in the eighth or ninth grade. This would result in leaving out much of the laborious introduction devoted to definitions and explanations. Comprehensive proofs of the theorems dealing with the areas of rectangles, parallelograms, triangles, and trapezoids would come under the same ban.

" . . . Because the early Greeks were led by their enthusiasm to prove everything in sight, it does not follow that present-day high school students should let loose their formal deductive logic whenever and wherever there is a chance to use it."

Thus we see the demon DEDUCTION haunting Mr. Nygaard again. He wants that demon exorcised. He certainly doesn't want it "let loose." Mr. Nygaard wants to avoid even the appearance of evil. He wants to abandon the "two column arrangement of the statements and reasons of a proof" which forces the poor student to become in some measure aware of the nature of the deductive method he is using. He gives an example of two forms of the same argument, electing the more "flexible" style as preferable. He urges a reason for his choice that "this second pattern has the further advantage in its close conformity to the style used in ordinary English composition."

Doesn't Mr. Nygaard realize that the first pattern which he condemns represents one of the first halting efforts of the race to free itself from the tyranny of words? Doesn't he realize the power in modern symbolism of which that formal style is a crude beginning? Doesn't he realize that the very reason which he urges in favor of the second "flexible" style is the evidence that it represents mathematical and logical retrogression and not progress?

I suggest that any one who is not clear as to the position I have taken here read just two books, both published by Macmillan and Company: *Fundamental Concepts of Algebra and Geometry* by J. W. Young and *Fundamentals of Mathematics* by M. Richardson. The first book is a sort of a classic which should be on the shelf of every teacher of mathematics in the schools. The latter book just happens to be one of the freshmen texts which we use here at Winthrop. I make this suggestion because I do not wish Mr. Nygaard or any of his audience to make the easy assumption that I am just an old fuddy-duddy with a nostalgia for the teaching

techniques of 1880 and a complex over such phrases as "inductive pragmatism." Mr. Young, Mr. Richardson, and myself are just as interested as Mr. Nygaard in improving the teaching of geometry and justifying its place in the school program.

But I wish to be sure that after "A Functional Revision of Plane Geometry" by Mr. Nygaard or any one else that what will remain will be plane geometry and not some queer hybrid subject consisting of a potpourri of unproved assertions, visual conjectures, and miscellaneous pep talks on the wonders of scientific method. Under Mr. Nygaard's plan who can be even reasonably sure that this is not just exactly what will happen? After all there is the disturbing thought that Mr. Nygaard would not really know whether what he had left was plane geometry or not. One's doubts are not dispelled either when one finds him saying:

"To the Greeks plane geometry was the big center of attraction, and was studied as a self-sufficient subject. Conditions are now entirely different. The technical, scientific achievements of modern civilization are a form of living mathematics that can not be ignored. There is probably as much geometry involved in Coulee Dam or the Golden Gate Bridge as in all the pages of Euclid. This applied geometry should be incorporated as thoroughly as possible into the teaching of the subject."

With the sentiment above expressed concerning the desirability of teaching applications in courses in mathematics few mathematicians would disagree. But I wish to draw your attention to the sentence "there is probably as much geometry involved in Coulee Dam or the Golden Gate Bridge as in all the pages of Euclid." This statement is essentially meaningless. What is "involved" in Coulee Dam is reinforced concrete and precious little else. What is *not* meaningless is some such statement as: there is considerable geometrical theory and more mathematical analysis in the planning of Coulee Dam than in all the pages of Euclid. You may believe that this is mere quibbling. I assure you that it is not, but reflects on the contrary a fundamental distinction in the attitude which one has towards the place of applications in mathematical instruction. More than once I have known progressive educationists to point to some mighty structure and say "that's living geometry." The proper retort to that statement is "that's hokum." But such "hokum" is unfortunately at times taken as a basis of instruction in geometry. It will be a long time before mathematicians will forget the spectacle of a progressive educationist leading a class around town making a cabalistic sign in the air faintly resembling a circle every time a member of the class observed a supposedly circular object such as a manhole cover or a door knob. This has been known to go on for days under the pretense of "studying the circle in real life situations." In reality this is nothing more nor less than the selling out of the heritage of the race by a bunch of salesmen who don't know what that heritage is.

I don't know whether Mr. Nygaard realizes the extent to which deductive logic permeates the entire structure of science. And as for philosophy there are at least some philosophers, notably Sir Arthur Eddington in *The Philosophy of Physical Science*, who even go so far as to assert that the entire structure of physics, even to the values of its fundamental constants, can be determined by deduction alone. However it may be with regard to this extreme position I do not know, but I did have the great good fortune to be trained first as an engineer and teach mathematics to engineers in one of the technical colleges of the country before beginning the more arduous task of training young teachers of mathematics; on the basis of this experience I assure Mr. Nygaard that Coulee Dam didn't get built of the Golden Gate Bridge get constructed by old Thales standing in front of the pyramids and mumbling to a bunch of ignorant Egyptians "that's living mathematics."

◆ THE ART OF TEACHING ◆

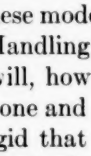
You Can Make Them

By CLARA O. LARSON
Taft High School, Chicago, Illinois

TO HANDLE things or to touch them gives a significance and concreteness that no amount of looking, drawing or thinking can give. For optional work in my plane geometry classes some of the boys made models, some from old erector sets and some from wood. Large models suitable for class demonstration at the board and small models suitable for seat work were made. Some of these models were:

1. Triangles. Handling a triangle shows, as nothing else will, how three sides definitely determine one and only one triangle and how very rigid that is. This fact naturally brings in examples of the use of the triangle in construction work, bridges, braces, etc. Models can be made of isosceles, equilateral, right, 30–60 degree, and 45–45 degree triangles.

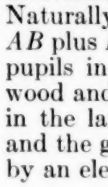
2. The quadrilateral which is not rigid.

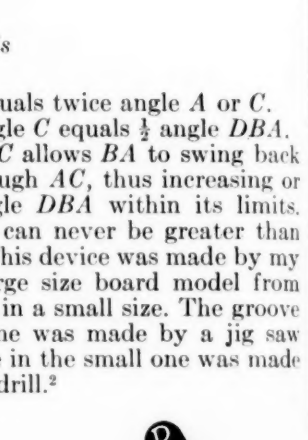


3. The quadrilateral with a diagonal bar which gives rigidity.[1]

4. The isosceles triangle which can be used as a device for bisecting an angle within the limits of the instrument.

AB equals BC
Angle A equals angle C
Angle DBA equals twice angle A or C.
Angle A or angle C equals $\frac{1}{2}$ angle DBA.
A groove in AC allows BA to swing back and forth through AC, thus increasing or decreasing angle DBA within its limits. Naturally AC can never be greater than AB plus BC. This device was made by my pupils in a large size board model from wood and also in a small size. The groove in the large one was made by a jig saw and the groove in the small one was made by an electric drill.[2]





5. A device for bisecting an angle.[3]

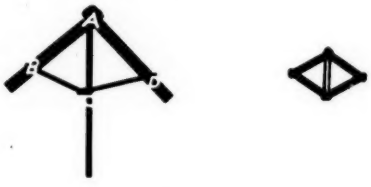
AB is made equal to AD
BC is made equal to CD
AC is equal to itself.
Therefore triangle ABC is congruent to triangle ADC and angles at A are equal. C swings up and down the bar AC, but at any time the triangles are congruent. One of my pupils made a sliding device of metal which allowed C to move up and down AC. Another one put a groove in the bar AC which allowed free motion. This device illustrates congruence and bisection of angle BAD.

[1] These ideas are based on exercises in "Essentials of Plane Geometry" by David Eugene Smith, page 34. By permission of the publishers, Ginn and Company.

[2] Prob. 6, page 166, "New Plane Geometry" by Stone and Mallory. By permission of the publishers, Benj. H. Sanborn and Co.

[3] Ex. 12 page 107, "Essentials of Plane Geometry" David E. Smith. By permission of the publishers, Ginn and Co.

Devices for bisecting an angle.

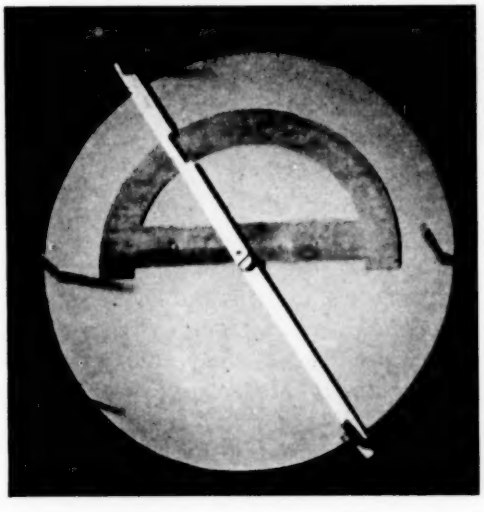


6. Parallel rulers.

These were easy to make. Small bolts and nuts were used to connect strips of plywood. The opposite sides were made equal. At all times in the free movements of the bars the figure is a parallelogram. One of my pupils used the parallel rulers to show that parallelograms having equal perimeters do not always have equal areas. By shifting the rulers, different parallelograms having equal perimeters can be drawn on squared paper. All these parallelograms will have the same bases but different altitudes. Since the area of a parallelogram is equal to the base times the altitude no two of these parallelograms will have the same area until the sides have passed through the perpendicular position. The rectangle had the greatest altitude, therefore the greatest area.

7. The boys who take mechanical drawing show the class how parallel lines are drawn by using the drawing board and a T square. Parallel lines are made because of the use of the geometric principle that two lines are parallel if the corresponding angles are equal.

8. The protractor with a sliding bar. Set on a tripod this very simple device can be used for sighting angles in the horizontal plane.



Models made and photographed by pupils of Taft High School.

An Echo from the Recent Annual Dinner of The National Council of Teachers of Mathematics at San Francisco

Said a youth of this new generation
Who refused to do multiplication
Please do not conclude
That I'm stupid or rude
It is just that I like motivation.

EDITORIALS

In Appreciation of President Potter

THE National Council of Teachers of Mathematics has grown considerably in recent years, not only in point of membership, but also in prestige and influence in educational affairs. For a long time Mary Potter has been a most loyal and valuable worker in the Council and she has had a large share in the growth and power of the organization. Since she has been president, however, the work of the Council has grown to such an extent that it places a heavy burden upon any one who has a full time position to fill besides the office of president. But this Miss Potter has done efficiently and graciously and at a great deal of personal sacrifice. The Council is indebted to her for years of conspicuous and important service.

W. D. R.

The New President of the National Council

DR. ROLLAND R. SMITH, Specialist in Mathematics for the Public Schools of Springfield, Massachusetts, was recently elected president of the National Council of Teachers of Mathematics. Dr. Smith has been very active in Council affairs for some time having just finished two three-year terms as a member of the Board of Directors.

Dr. Smith, a native of Massachusetts, graduated from the Classical High School in Springfield where he has since taught during most of his career. He received his A. B. degree at Harvard in 1915 with *magna cum laude* in mathematics and education. He is also a member of Phi Beta Kappa and Phi Delta Kappa. He taught mathematics in the Classical High School in Springfield from 1915 to 1921 and in the Classical High School at Newtonville, Massachusetts from 1921–1925. He was Assistant Professor of Mathematics in Education at Clark University at Worcester, Massachusetts from 1925 to 1927. He received the Ed. M. degree from Harvard in 1927. He then returned to Springfield as head of the mathematics department of Classical High School where he has been ever since.

Dr. Smith received his Ph. D. degree from Teachers College, Columbia University in 1940. He has given courses in mathematics at Teachers College since 1930. He is co-author of several books on mathematics including texts and a book on the teaching of mathematics. His Ph. D. dissertation dealt with pupils' errors and the development of methods to overcome them. He has lectured widely and is equipped as few teachers are to discuss matters pertaining to mathematical education. Dr. Smith is married and lives at Longmeadow, Massachusetts.

We congratulate Dr. Smith on his election to the presidency and wish him success in every way. His task will not be an easy one in the trying days ahead, but he is well qualified to carry on the good work done by President Potter. We congratulate the Council in having the services of so well qualified a teacher and leader as Dr. Smith.

W. D. R.

Message from the New President

To the officers, members, and friends of the National Council of Teachers of Mathematics:

I fully appreciate the honor of being made your president, but now that the first excitement is over, my feeling is one of humility toward the responsibility you have placed upon me. Just now a generation which has sometimes held mathematical education in contempt is forced to see its value. We must not take this opportunity as an easy way out. We cannot afford to be war profiteers. There is still much to be done in improving the teaching of mathematics. Pioneers in the National Council have shown us the way. We must not fail them.

There is also a larger sense in which we must not fail. We shall win this physical war somehow, sometime, but we must win the moral war. Teachers will be asked to sacrifice of their finances, their time, and their energy. One thing they must not give up. They must show the way toward developing in our young people the ideals and responsibilities of our democracy so that the tragedy of these days may not happen again. "The truth shall make you free."

Rolland R. Smith

Concerning Subscriptions That Expire in May

ANY ONE who began his subscription to THE MATHEMATICS TEACHER with the October issue in 1941 is automatically a member of The National Council of Teachers of Mathematics until October, 1942. However, since no issues of the magazine are published in June, July, August, and September of 1942, those who paid the membership fee of $2.00 in October of last year should send in their renewals before October, 1942 in order to save the Council inconvenience and loss of money. Costs of publication are rising and in order not to have to raise the price of the journal during the emergency we bespeak the cooperation of our members in being prompt in making renewals. In order to make sure that this matter is not overlooked, THE MATHEMATICS TEACHER will send out early in April, if not before, cards to all members whose subscriptions expire in May (even though their membership runs to October) and it is hoped that members will be prompt in renewing membership so as to facilitate matters in the office. It has been almost impossible to plan for the October issue each year because members are so careless about renewing in time. Moreover, entirely too many members fail to renew at all even though two or three notices and a personal appeal from the Editor have been sent out. It is more important now than ever before that every present member "stick by the ship" if we are to weather the storms ahead and if the Council is to continue to do its work effectively.

W. D. R.

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

American Mathematical Monthly

January 1942, Vol. 49, No. 1

1. MacNeish, H. F., "Four Finite Geometries," pp. 15–23.
2. Coe, C. J., "Problems on Maxima and Minima," pp. 33–37.
3. Fort, Tomlinson, "Summability and the Definition of a Limit," pp. 37–43.
4. McIntyre, D. P., "A New System for Playing the Game of Nĭm," pp. 44–46.
5. Blanche, E. E., "A Night with Probability," pp. 54–60.

Bulletin of the Kansas Association of Teachers of Mathematics

April 1941, Vol. 15, No. 4.

1. Altshiller-Court, Nathan, "Democratizing Mathematics," pp. 55–57.
2. Zant, James H., "General Mathematics for College Students," pp. 57–60.
3. Abraham, C., "Considerations on the Teaching of General Mathematics to Pre-Professional College Students," pp. 60–61
4. Matthews, James A., "Teaching of Genera. Mathematics in Grade Nine," pp. 61–62.
5. Wilson, Mary Mason, "A Method of Teaching Verbal Problems," pp. 62–64.
6. Smutz, F. A. "The Value of Solid Geometry," pp. 64–65.
7. Sperry, A. B., "Mathematics in Geology," pp. 65–66.
8. Hart, William L., "Mathematics and National Preparedness" (abstract of address), p. 67.
9. Wren, F. L., "The Role of Relational Thinking in the Teaching of Secondary Mathematics" (excerpts from address), pp. 67–68.
10. Cecil B., "Helping the Student Choose a Slide Rule," p. 68.

National Mathematics Magazine

January 1942, Vol. 16, No. 4.

1. Yates, Robert C., "The Trisection Problem" (chap. 5). pp. 171–182.
2. Finkel, Benjamin F., "A History of American Mathematical Journals" (Continued), pp. 188–197.
3. Sleight, E. R., "Early English Arithmetics," pp. 198–215.

School Science and Mathematics

February 1942, Vol. 42, No. 2.

1. Potter, Mary A., "Trends in Teaching Mathematics," pp. 109–115.
2. Jackson, William N., "What Can Be Done in Algebra?," pp. 142–156.
3. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 174–177.
4. Volpel, Marvin C., "Results of an Inventory Test," pp. 188–189.

Miscellaneous

"Advanced Mathematics and the Problem of Defense," *School and Society*, 54: 5–6, July 5, 1941.

Barnhart, W. S., "Problems and Issues with Reference to Teaching Procedures in Business Arithmetic," *Seventh Yearbook of the National Business Teachers Association*, pp. 227–231.

Brownell, W. A., "Contrasting Views of Arithmetic in the Elementary Curriculum," *University of Pennsylvania School of Education: Coordination of School and Community*, pp. 392–396.

Buswell, G. T., "Selected References on Elementary-School Instruction: Arithmetic" (continued), *Elementary School Journal*, 42: 212–215, November, 1941.

Durell, T. J., "Developing an Understanding of Arithmetic," *University of Pennsylvania School of Education: Coordination of School and Community*, pp. 384–392.

Greenfield, E. V., "Mathematics and Foreign Languages—Tough Hombres," *Modern Language Journal*, 25: 703–710, October, 1941.

Graham, P. H. "Student Who Found Himself in Mathematics," *School and Society*, 54: 249–250, September 27, 1941.

Grossnickle, F. E., "Types of Errors in Division of Decimals," *Elementary School Journal*, 42: 184–194, November, 1941.

Johns, W. B., "What About Secondary Mathematics?", *Nebraska Educational Journal*, 21: 268+, October, 1941.

Knight, F. B., "Fundamentals in the Arithmetic Program," *Proceedings National Educational Association, 1941*, p. 128.

Lahey, M. F. L., "Permanence of Retention of Retention of First year Algebra," *Journal of*

Educational Psychology, 32: 401–513, September, 1941.

Nelson, A. E., "Certain Skills Must Be Mastered: Mathematics," *School (Elementary Edition)*, 30: 342–345, December, 1941.

Petrie, P. A., "Teaching a New Topic in Mathematics," *School (Secondary Edition)*, 30: 59–62, September, 1941.

"Report of the Subcommittee in Education for Service of the American Mathematical Society and the Mathematical Association of America," *Science*, 94: 205, August 29, 1941.

Russel, M. B., "Number Test," *Instructor*, 51: 17, January, 1942.

Sands, L. B., "Providing for Arithmetic Experiences in the Elementary School," *Thirteenth Yearbook of the California Elementary School Principals Association*, pp. 79–82.

Sister Mary Henrietta, "Your Problem and Mine: Mathematics," *School (Elementary Edition)*, 30: 245–250, November, 1941.

Smithline, H. M., "Relating Business Arithmetic to the Everyday Needs of the Future," *Fourteenth Yearbook of the Eastern Commercial Teachers Association*, pp. 129–132.

Van Engen, H., "Unifying Ideas in Arithmetic Instruction," *Elementary School Journal*, 42: 291–296, December, 1941.

Wheeler, L. R., "Comparative Study of the Difficulty of Learning the Multiplication Combinations," *Pedagogical Seminary and Journal of Genetic Psychology*, 59: 189–206, September, 1941.

The Intrinsic Harmony of Number

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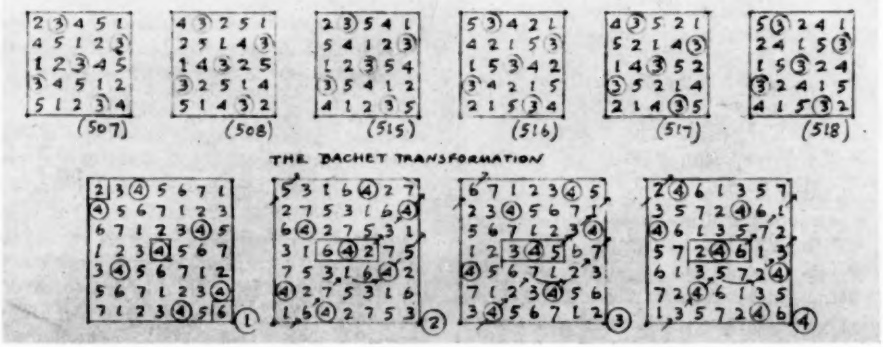

With the first two primary squares (507) and (508)—each used in its eight forms—the complete set of 16 associated—Nasik magic squares of the fifth order may be formed. These are illustrated for the first time in the Second Paper. The rules by which Frierson's set of 3600 Nasik squares may be formed with combinations of the above six primary squares are also given.

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BOOK REVIEWS

The Philosophy of Albert North Whitehead. Edited by Paul Arthur Schilpp. Vol. III in "The Library of Living Philosophers," Northwestern University, Evanston and Chicago, 1941, xx and 745 pp. Price, $4.

In this interesting new book the "unique philosophical contributions and ideas" of Professor Whitehead are "described and critically appraised by some of his greatest philosophical contemporaries and other prominent scholars." There are eighteen contributors in all. Professor Whitehead himself contributes (1) his own autobiography and (2) two previously unpublished essays on "Mathematics and the Good" and on "Immortality," which together constitute what Whitehead calls a sufficient "Summary" and concluding statement of his philosophical position; (3) A facsimile reproduction of an explanatory statement in Whitehead's own handwriting penned specifically for this volume is included.

Those who have followed the long career of Professor Whitehead both abroad and in this country are going to appreciate the appearance of this important volume. It deserves a large audience.

W. D. R.

Early Military Books in the University of Michigan Libraries. By Thomas M. Spaulding and Louis C. Karpinski. The University of Michigan Press, Ann Arbor, xvi and 371 pages. Price, $2.

This new book will be of value to all those who are interested in the history of military affairs. It is well made and attractively printed. It is "a list or guide, often with condensed titles, indicating Michigan's resources, and not a bibliography or even an attempt at one."

"Early" books include those published before or during the year 1800; later editions are omitted, even though the book first appeared before the limiting date.

By "military" books is meant those dealing with the military science and art; military history, as such, is omitted. Works on military medicine are not listed. Military law is included. A large number of books listed are primarily mathematical.

W. D. R.

Natural Trigonometric Functions. By Leo Hudson and E. S. Mills. Monroe Calculating Machine Company. Price, $6.

This book is the answer to a need for eight-place tables of natural trigonometric functions. It gives an approach to the solution of problems in trigonometry by a rapid method without the use of logarithms.

The style of arranging the values is a special feature for all the functions of each degree are given on a single page. Thus the factors which otherwise require a large, cumbersome volume, or even several volumes, are condensed into a neat handbook of less than seventy pages. That makes reference quick and easy, saving lots of thumbing over pages. The size is handy for office and drafting room, convenient for carrying in the pocket or brief case when in the field.

Detailed methods of using the tables with a calculating machine are clearly explained in simple language and with illustrations. There are also several pages of helpful formulas and practical short-cuts for solving right and oblique triangles.

W. D. R.

The Pythagorean Proposition. by Elisha S. Loomis. Baldwin-Wallace College, Berea, Ohio. xi and 285 pages. Price, $2.

This book is a final revision of a previous volume known to many teachers of mathematics. The object of this book is "to present to the future investigator, simply and concisely," what is known relative to the so-called *Pythagorean Proposition* (known as the forty-seventh proposition of Euclid and as "Carpenter's Theorem"), and "to set forth certain established facts concerning the algebraic and geometric proofs and the geometric figures pertaining thereto."

This is a book that should be in every high school library and all teachers of mathematics will find it most interesting and helpful.

W. D. R.

Algebraic Solid Geometry. By S. L. Green. Macmillan. 132 pp. Price, $1.75.

This new book is based upon a course of lectures given, during the past twenty years, to students of Queen Mary College "reading for

Pass, and General and Special Honours degrees in Arts, Science and Engineering." It presupposes a knowledge of elementary algebra including determinants.

The chapter headings are as follows:

1. Rectangular cartesian coordinates: Direction Cosines of a Line.
2. The Plane and The Straight Line.
3. The Sphere.
4. The Central Quadrics.
5. The Paraboloids.
6. The Cone.

W. D. R.

An Introduction to Analytical Geometry. By A. Robson. Cambridge at the University Press. 1940. xiv and 409 pp.

This book furnished an introduction to the use of coordinates and analytical methods in geometry. It is expected that a student who is to read the book will ordinarily have taken a previous course in elementary calculus and that from his study of graphs he will have obtained some knowledge of the cartesian coordinate system. However, a previous course in coordinate geometry is not essential, since the earlier parts of the book contain numerous exercises for one who has had no previous training.

A distinction between metrical and projective geometry was made by the author in preparing the book so as to form a basis for later University work.

W. D. R.

An Experiment in the Development of Critical Thanking. By Edward M. Glaser. Teachers College, Columbia University Contributions to Education, No. 843, Bureau of Publications, ix and 212 pages. Price, $2.35.

This book is a report on the experimental study of the ability to think critically. The ability to think clearly as conceived in this volume involves three things:

1. An attitude of being disposed to consider in a thoughtful way the problems and subjects that come within the range of one's experiences.
2. Knowledge of the methods of logical inquiry and reasoning.
3. Some skill in applying these methods.

The study is concerned with the problems of how to educate pupils more effectively for responsible, competent citizenship in a democracy. The author describes and analyzes an experiments in which five different classes of high school students were taught for a ten-week period with specially developed materials and methods, while five equated groups of students followed the usual course of study.

The six types of evaluation procedures (including "living documents" from the students and teachers involved) all showed that the groups receiving the special training made significant gains in ability to think critically and reason logically.

The comprehensive review and bibliography of previous research in the psychology, training, and measurement of critical thinking ability make this volume a useful reference book for students in the field.

W. D. R.

A First Course in Trigonometry. By W. J. Oliver, P. F. Winters and J. E. Campbell. School Aids and Text Book Publishing Co., Ltd. Regina and Toronto. 269 pages. Price, $1.25.

This text differs from many others in that the numerical trigonometry part precedes the analytical treatment of this subject. This is intended to make the book easier and more interesting because of applications to real problems that can be understood by the student.

The book is divided into three parts as follows:

Part I. Numerical Trigonometry.
Part II. Analytical Trigonometry.
Part III. Tables.

The material of the book is flexible and may be arranged to suit the special needs of a given group.

There are a large number of exercises in the book, thus affording the teacher a chance to make a careful selection suited to his needs.

W. D. R.

Mathematics for the Aviation Trades. By James Naidich. McGraw-Hill. x and 267 pp. Price, $1.80.

This book was written for students in trade and technical schools who intend to become aviation mechanics. It is intended to satisfy the demand on the part of instructors and employers that mechanics engaged in precision work have a thorough knowledge of the fundamentals of arithmetic applied to their trade.

Each new topic is presented as a job most of which can be covered in one lesson.

The book covers a review of fundamentals for the airplane mechanic, the airplane and its wing, mathematics of materials, aircraft engine mathematics and a final review chapter.

W. D. R.

Elementary Statics. By M. Appleby. Cambridge at The University Press. 1939. 164 pp.

This book is based on a series of lectures given to Engineering students and does not, therefore, claim to have developed the subject "rigorously from a minimum number of funda-

mentals." It treats forces from the outset "as acting on a body, and not as mere abstractions."

Graphical methods are used on practical examples and in other ways the work is simplified. After an introductory chapter, there are other short chapters on translation and rotation, work and machines, friction, center of gravity, graphical statics, couples and pure rotation, and some miscellaneous material.

Teachers interested in teaching some mechanics in the schools will find this work of interest.

W. D. R.

Adventures in Algebra. Second Course. By Murray J. Leventhal, Charles Salkind and F. Eugene Seymour. Globe Book Co., 1941. xiv and 493 pp.

This new book is intended to break down the barriers between the various compartments of mathematics as much as possible in line with modern demands for more correlation. It proposes to unify the course by making the function concept the unifying thread.

The book is planned so that it can be used for one or two semesters as local situations demand. There is ample problem material, but a balance is struck between the practical and the analytical aspects of the subject.

The book will meet the needs of all those who wish to meet extra-mural requirements and it will be of interest to all teachers of intermediate algebra, like the authors, who know what mathematics is necessary to meet these requirements.

Intermediate Algebra. By H. L. Rietz, A. R. Crathorne and L. J. Adams. Henry Holt and Co. 1942. viii and 248 pp. Price, $1.75.

This new book is intended for those schools which offer a one-semester course in intermediate algebra to students who have studied only elementary algebra.

The book is based upon classroom experimentation done by the authors who claim to have selected practical and carefully graded problem material.

There is not a great amount of explanatory material, the book being mostly problems and exercises. Whether this is a good plan or not depends largely upon the point of view of the one who uses the book.

W. D. R.

The Teaching of Secondary Mathematics. By Chas. H. Butler and F. Lynnwood Wren. McGraw-Hill Gook Co. 1941. xii and 513 pages. Price, $3.00.

This is a very timely new book which every teacher of secondary mathematics should put on his must list for reading. It should be read also by general educators and supervisors who have any authority over the work in mathematics in secondary schools. Written by two experienced and capable teachers, the book is helpful and inspirational, particularly to the group of younger teachers who, lacking experience in the classroom, need guidance in their daily lesson plans.

The book is divided into three parts:

1. The Place and Function of Mathematics in Secondary Education.
2. The Improvement and Evaluation of Instruction in Secondary Mathematics.
3. The Teaching of the Special Subject Matter of Secondary Mathematics.

The authors are to be congratulated on having done an excellent piece of writing and while one may not agree with every detail of philosophy or suggestions for the improvement of instruction, it is sane and well balanced.

The exercises and the bibliographies at the end of each chapter are well chosen and timely.

The book deserves a large sale among teachers who have been looking for just such a book.

W. D. R.

What is Mathematics? By Richard Courant and Herbert Robbins. Oxford University Press. 1941. xix and 521 pages. Price, $3.75.

This is a most interesting and important addition to the literature on mathematical education. It should be read by every teacher of secondary mathematics who wishes to have a background for his instructional work in the schools.

After an interesting introduction on "What is Mathematics?," the authors have presented eight chapters and an appendix. Some of the chapters have supplements and appendices. The main chapter headings run as follows:

1. The Natural Numbers.
2. The Number System of Mathematics.
3. Geometrical Constructions. The Algebra of Number Fields.
4. Projective Geometry. Axiomatics, Non-Euclidean Geometrics.
5. Topology.
6. Functions and Limits.
7. Maxima and Minima.
8. The Calculus.

The book is attractively made, well illustrated, and contains a well of material for those who wish to know a little more than they are required to teach to their pupils. The book is clearly written and should be easy for teachers of secondary mathematics to read.

W. D. R.